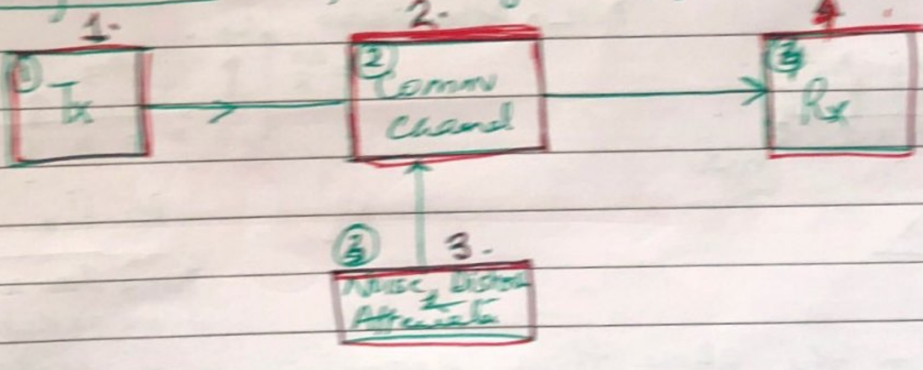
LECTURE # 05 - Topic 2 (Contd)

Recap & Review of Previous Lecture

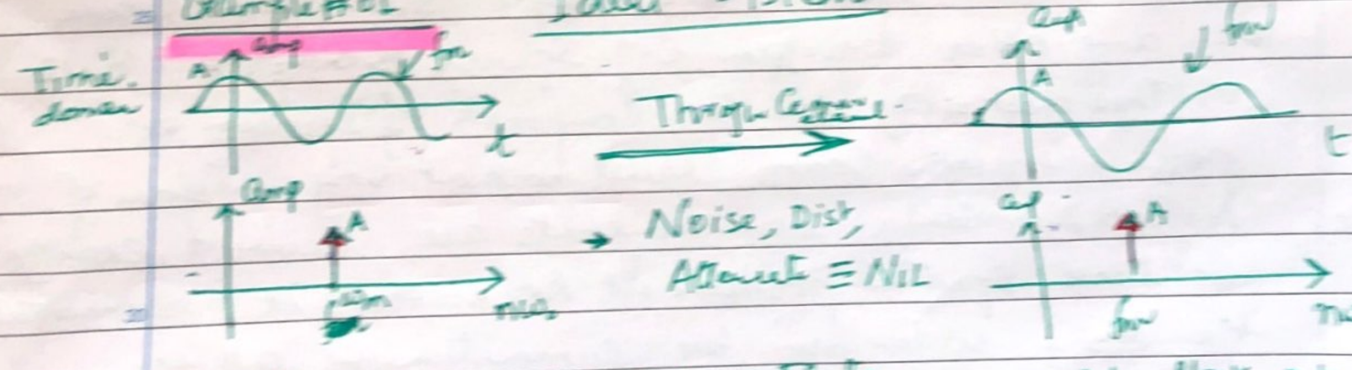
Looking back at the basic Electronic Communication block diagram [Fig 2.1; pg 15; dt: 21.04.20] we shall see how a signal changes both in Time domain and Frequency domain through Amp-time plots & Amplitude-frequency plots respectively.
We shall try to make use of such plots established by Fourier Series Expansion in previous lectures.
Also we need to understand the use of such analysis in Comm. System

Q. So what do you infer and establish from the Fourier Series Expansion as applied to various questions [Q 1 - Q.16] [Tsheet #02] ?

Ans. Some Example 1. Refer → Fig 2.1; pg 15; dt 21.04.20.



Example #01 Ideal System

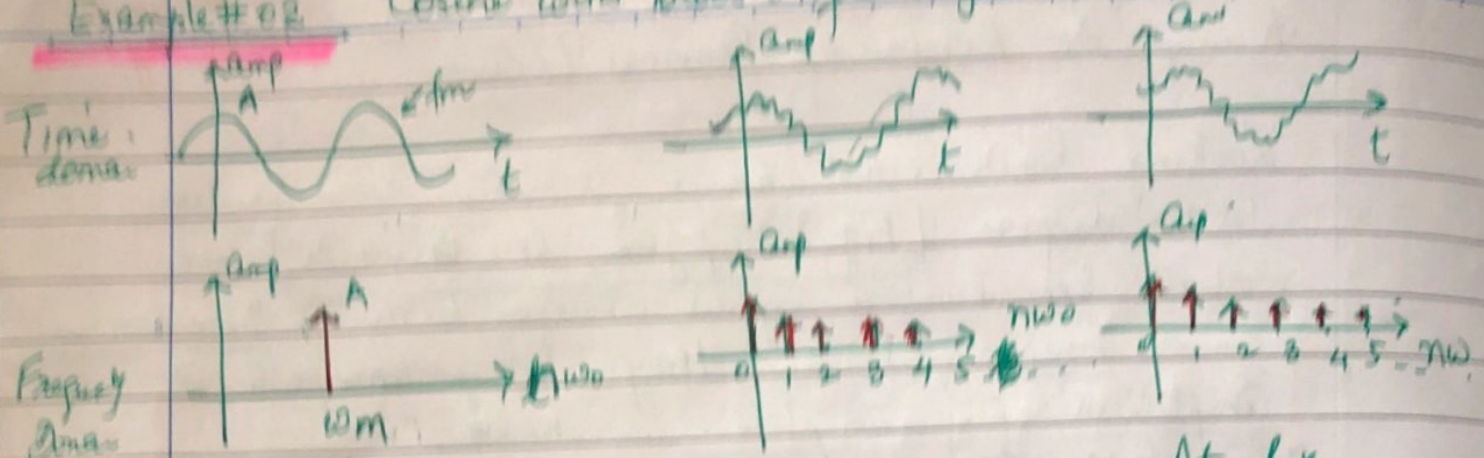


At Block 01 → No contribution of Block 3 in Block 2 At Block 04

At Tx At Rx.

Time domain at Rx is same & identical of Tx

Example #02 Cosine with Block & influence of Noise, Distortion & Attenuation

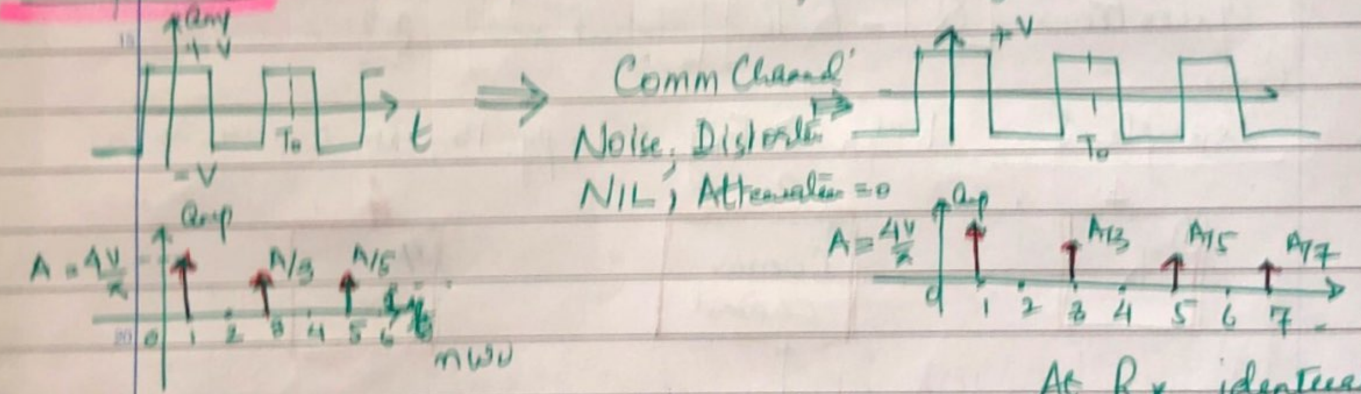


AT Tx
Single fundamental Component at n=1

→ Noise & Distortion Picked up by Comm. Channel.
→ Signal Attenuated

At Rx
→ Distortion of Signal Received
→ Now Signal comprises of infinite components.

Example #03 Square periodic waveform through ideal Comm Channel

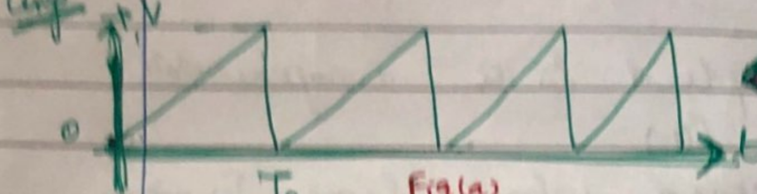


At Tx

At Rx identical time domain signal same as that of Tx

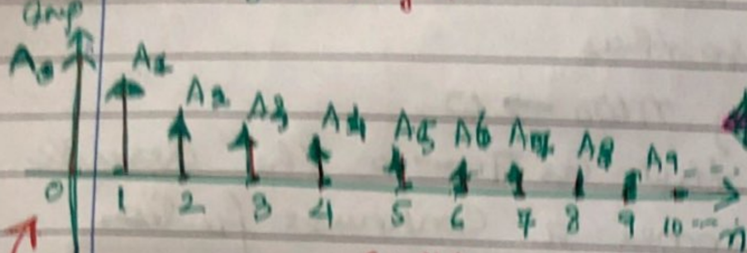
So from the above three examples we understand that in ideal systems be it single sinusoid or Cosine or any other waveform e.g Square of example #03 (Q a 1(b) of Theory sheet #02) all the Components of Spectrum are passed through Comm: Channel without any Interference of Noise & Distortion. Same Components are received exactly at same spectral positions at Rx so that it reconstructs the same waveform by summation of all the oscillating Cosines with their respective harmonically related frequencies.

Example # 04 **In General: look at the example below**



Fig(a)

Time domain Signal as Generated by Tx

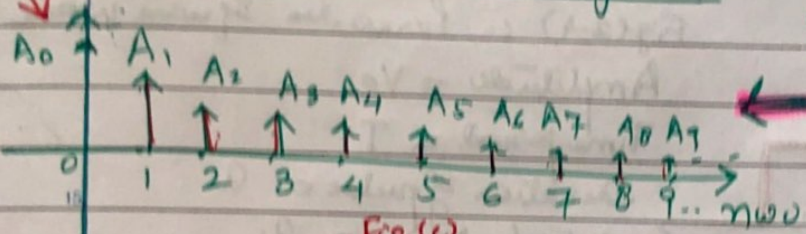


Fig(b)

After fourier Series
This is the Spectrum.
$A_0 \rightarrow$ dc Component
$A_1, A_2, A_3 \ldots A_\infty$
are oscillating cosines

identical Spectrum

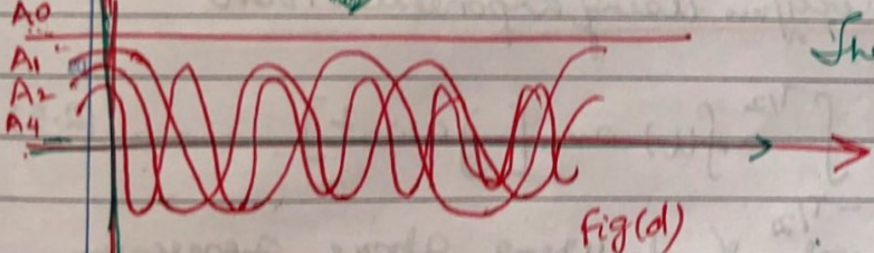
IF ideal comm. System



Fig(c)

Identical Spectrum received if there is no influence of Noise & Distortion
→ Fig(b) & Fig(c) are identical.

After Reconstruction at R_x of Spectrum received (Fig c).



Fig(d)

The Rx reconstructs each component as received of Fig(c) Spectrum

This is as good as imagining at Rx Summation of $[A_0 \rightarrow$ dc component $\omega_0 = 0]$ + [Oscillating Cosine of Amp A_1 at ω_0]
+ [Oscillating Cosine of Amp A_2 at $2\omega_0$] + [Oscillating Cosine of Amp A_3 at $3\omega_0$].
— — — — — — + [Oscillating Cosine of Amp A_n at $n\omega_0$]

Result will be Reproduction of same time domain signal of Fig (a).

Now if Comm. Channel has influence of distortion & Noise then At receiver Spectrum of Fig(b) will not match the received Spectrum of Fig(c). Result will be that Reproduced & reconstructed time domain will not match that of Fig(a) Time domain

2.5 Fourier Integral & Fourier Transform

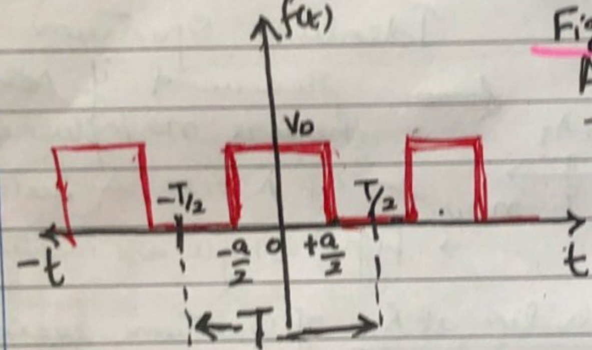
→ Fourier Integral is applied to a non-periodic (aperiodic) function $f(t)$

→ Fourier Integral is used when time $T_0 \to \infty$ in Fourier Series so that

$$\text{as } T_0 \to \infty \; ; \; n\omega_0 \to \omega$$

Because $\frac{2\pi}{T_0}$ shows that as $T_0 \to \infty$ the discrete spectrum then becomes a continuous function

Take once again the example of a square periodic wave below.



Fig(2.4) :- Periodic Square wave
Amplitude = V_0
Time period = T
Duration of pulse = a
Duty factor = $\frac{a}{T}$

2.5.1 Evaluating Fourier Coefficients using Exponential Form

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) \exp[-jn\omega_0 t] \quad \text{- - - (2.5)}$$

Here C_n is evaluated using above expression

$$C_n = \frac{V_0}{\pi n} \left[\sin(n\omega_0 a/2) \right] \quad \text{- - - (2.6)}$$

C_n represents the amplitude of n^{th} discrete component.

→ Here our purpose is to find how the C_n amplitude changes as $\frac{a}{T}$ duty factor changes in a periodic fn

→ So to say as $\frac{a}{T}$ changes the discrete frequency spectrum of time domain for $f(t)$ of Fig 2.4 also changes.

Divide numerator & denominator of Eq (2.6) by $n\omega_0 a/_2$

$$C_n = \frac{V_0}{\pi n}\left[\frac{\sin(n\omega_0 a/_2)\cdot n\omega_0 a/_2}{n\omega_0 a/_2}\right]$$

Using $\omega_0 = \frac{2\pi}{T}$; then above Eqn can be rewritten as:

$$\boxed{C_n = V_0 . \left(\frac{a}{T}\right)\left[\frac{\sin(n\omega_0 a/_2)}{n\omega_0 a/_2}\right]} \quad \ldots (2.7.a)$$

OR

$$C_n = V_0\left(\frac{a}{T}\right)\left[\frac{\sin\left(n\frac{2\pi a}{2T}\right)}{n\frac{2\pi a}{2T}}\right]$$

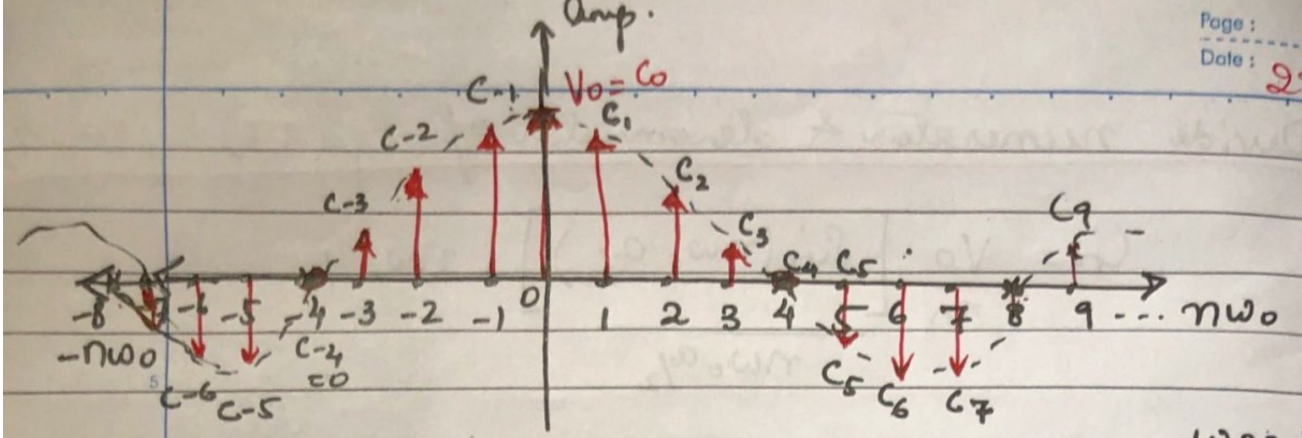
OR

$$\boxed{C_n = V_0\left(\frac{a}{T}\right)\left[\frac{\sin(n\pi a/_T)}{n\pi a/_T}\right]} \quad \ldots (2.7.b)$$

C_n of Eq (2.5) or equivalently of Eq (2.7.a) or Eq (2.7.b) represents the coefficient of exponential form of Fourier Series expansion given by (2.8) eqn below

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} C_n \exp[jn\omega_0 t]} \quad \ldots (2.8)$$

↔ The above Eqn represents the Discrete Double Sided Amplitude Spectrum which looks as in illustration of Fig (2.5)



$\omega_0 = \frac{2\pi}{T}$

Fig [2.5] :- Double sided Descrete spectrum of a periodic square wave. Spacing b/w each Equispaced component is $\Delta\omega = (n\omega_0) - (n-1)\omega_0$

$$= n\omega_0 - n\omega_0 + \omega_0$$

$$\Delta\omega = \omega_0 = \frac{2\pi}{T}$$

→ C_0 is the dc component for $\omega_0 = 0$ position.

→ Now as $T \to \infty$ ∴ $\boxed{n\omega_0 \to \omega}$

Discrete scale — continuous scale

→ $C_{+1} = C_{-1} = V_0 \frac{a}{T} \frac{\sin(\omega_0 a/2)}{\omega_0 a/2}$

$C_{+2} = C_{-2} = V_0 \frac{a}{T} \frac{\sin(2\omega_0 a/2)}{\omega_0 a/2}$

\- - - - - - -

$C_{+n} = C_{-n}$

→ Some Components may be Zero (eg in figure $n = 4, -4$)
Some Components may be +ve (eg $n = \pm 1, 2, 3, 9, 10 \ldots$)
Some Components may be −ve (eg $n = \pm 5, 6, 7$).

→ Positive Component Values Correspond to a phase $\phi_n = 0$

→ Negative Component Values Correspond to a phase $\phi_n = \pi$

→ C_n is a discrete function of $n\omega_0$.

→ Hence Equation (2.7.a) or (2.7.b) has values only for discrete frequencies $n\omega_0$ and each component is equispaced by $\Delta\omega = \omega_0 = \frac{2\pi}{T}$.

2.5.2. Examination of C_n as duty factor $\left(\frac{a}{T}\right)$ changes :-

Again we may write

$$C_n = V_0\left(\frac{a}{T}\right)\left[\frac{\sin(n\omega_0 a/2)}{n\omega_0 a/2}\right] \quad \ldots (2.7.a)$$

OR

$$\boxed{C_n = V_0\underbrace{\left(\frac{a}{T}\right)}_{\text{duty factor}} \frac{\sin\left(n\pi\overbrace{\left(\frac{a}{T}\right)}^{\text{duty factor}}\right)}{n\pi\underbrace{\left(\frac{a}{T}\right)}_{\text{duty factor}}} \quad \ldots (2.7.b)}$$

We can observe the changes in discrete spectrum for

(i) $\frac{a}{T} = \frac{1}{2} \Rightarrow T = 2a$

(ii) $\frac{a}{T} = \frac{1}{4} \Rightarrow T = 4a$

(iii) $\frac{a}{T} = \frac{1}{5} \Rightarrow T = 5a$

(iv) Generally $\frac{a}{T} = \frac{1}{N} \Rightarrow T = Na$.

(i) $\frac{a}{T} = \frac{1}{2} \Rightarrow T = 2a$ substitute in Eq (2.7.b)

Eq 2.7.b is modified as

$$C_n = \frac{V_0}{2}\frac{\sin\left(\frac{n\pi}{2}\right)}{\left(n\pi/2\right)} \quad \ldots (2.8.a)$$

(b) $\frac{a}{T} = \frac{1}{4} \Rightarrow T = 4a$ Then Eqn (2.7.b) becomes

$$C_n = \frac{V_0}{4} \frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi/4} \quad \ldots \text{Eqn } (2.8.b)$$

(C) $\frac{a}{T} = \frac{1}{5} \Rightarrow T = 5a$, Then Eqn (2.7.b) becomes

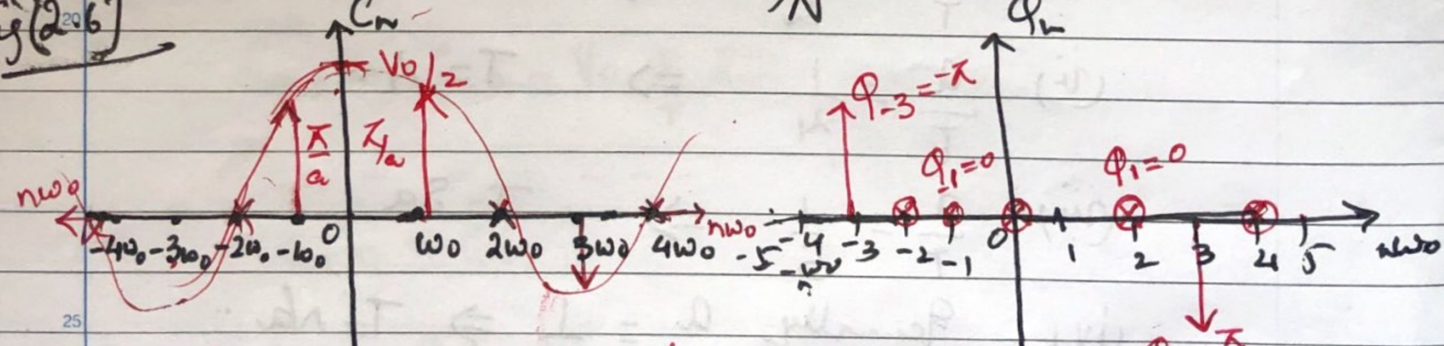
$$C_n = \frac{V_0}{5} \frac{\sin(n\pi/5)}{n\pi/5} \quad \text{Eq } (2.8.c)$$

(d) finally in generall

$$\frac{a}{T} = \frac{1}{N} \Rightarrow T = Na$$

$$C_n = \frac{V_0}{N} \frac{\sin(n\pi/N)}{n\pi/N} \quad \text{Eq } (2.8.d).$$

Fig (2.6)



Frequency spectrum → Even symmetry
→ Discrete Components
→ Some +ve, Some −ve, Some Zero Values
→ Peak Value is $V_0/2$.
→ $\Delta\omega = \frac{2\pi}{T_0} = \frac{2\pi}{2a} = \pi/a$

Phase Spectrum: Odd symmetry
→ Discrete Components
→ +ve Values $\equiv \varphi_n = 0$
∴ −ve Values $\equiv \varphi_n = +\pi$ on one side
$\varphi_{-n} = -\pi$ on another side

So as T increases as in Case (b), (c), (d) then $\Delta\omega$ also reduces.

Similarly we can plot Amplitude and phase spectrum corresponding to case (b, c, d for T = 4a, ~~T=~~ T = 5a etc).
[Refer Van Valkenburg for these examples of T=2a; T=4a, T=5a, T=Na ~~~~ to be attempted ~~as~~ in Tsheet].

Conclusion: N larger means T is larger.

→ In general $\Delta \omega = \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{aN}$

As N becomes larger, $\Delta\omega$ becomes smaller so that the discrete ~~~~ term $n\omega_0 \to \omega$

→ Amplitude of comp. at $n=0$ is V_0/N.
As N becomes larger V_0/N becomes smaller.

→ The number of lines between (Zero Crossing) is $(2N-1)$.
As $N \to$ ~~l~~arger ∴ Spacing b/w discrete components is also large. So that discrete spectrum becomes Continuous spectrum.

→ Spectrum envelope for a non recurring pulse is a continuous function found by replacing $n\omega_0$ by ω. Envelope Traced out is

$$= V_0 \frac{a}{T} \frac{\left(\sin \omega a/2\right)}{\omega a/2}$$

Tsheet # 02 (Contd).

Q.17 Draw the double sided Amplitude spectrum & phase spectrum of a square periodic pulse of Fig[2.4] on pg 45 for (a) $\frac{a}{T} = \frac{1}{2}$, (b) $\frac{a}{T} = \frac{1}{4}$; (c) $\frac{a}{T} = \frac{1}{5}$; (d) $\frac{a}{T} = \frac{1}{8}$
[Refer Van Valkenburg].

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