→ According to n/w theory a n/w can be described by a no of parameters like Z, Y, H & ABCD.

a) Z parameter :→ $V = IZ$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

b) Y - parameter :→ $I = VY$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

c) H- parameters :→

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

d) ABCD parameter :→

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Scattering Parameters :→ As we move to high freqs like microwave freqs circuits network parameters can't be used like h, y and z parameters. The reasons being :—

i, Equipments are not available to measure total voltage & current at the port of n/w

ii, Short ckt & open ckt are difficult to achieve over broad band & high freqs

iii, Active devices like tunnel diodes etc will not be stable at open & short ckts.

So, for μ-wave freqs we have another set of parameters called S-parameters

$$S_{ij} = \frac{\text{Normalized ref. wave at } i^{th} \text{ port}}{\text{Normalized incident wave at } j^{th} \text{ port}}$$

i → ref (dest)

j → incident (source)

$$S_{ij} = \frac{b_i}{a_j}$$

b_i → Normalized ref wave (volt/cu

a_j → Normalized inci wave (vol/c

Norm. ref. voltage wave = $\frac{\text{ref wave}}{\sqrt{Z_0}}$

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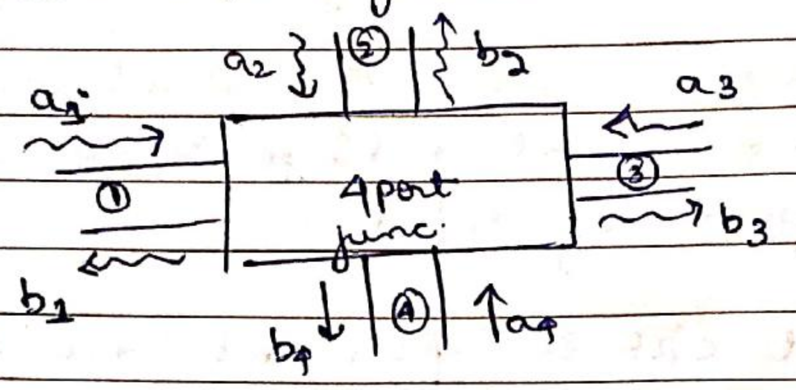
IIly $a_j = \dfrac{V_j^+}{\sqrt{Z_{0j}}}$

for a 4-port junc :→



$S_{11} = \dfrac{b_1}{a_1}$ $\quad S_{12} = \dfrac{b_1}{a_2}$ $\quad S_{13} = \dfrac{b_1}{a_3}$ $\quad S_{14} = \dfrac{b_1}{a_4}$

$S_{21} = \dfrac{b_2}{a_1}$ $\quad S_{22} = \dfrac{b_2}{a_2}$ $\quad S_{23} = \dfrac{b_2}{a_3}$ $\quad S_{24} = \dfrac{b_2}{a_4}$

$S_{31} = \dfrac{b_3}{a_1}$ $\quad S_{32} = \dfrac{b_3}{a_2}$ $\quad S_{33} = \dfrac{b_3}{a_3}$ $\quad S_{34} = \dfrac{b_3}{a_4}$

$S_{41} = \dfrac{b_4}{a_1}$ $\quad S_{42} = \dfrac{b_4}{a_2}$ $\quad S_{43} = \dfrac{b_4}{a_3}$ $\quad S_{44} = \dfrac{b_4}{a_4}$

$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4$

IIly $b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4$

$b_3 =$

$b_4 =$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

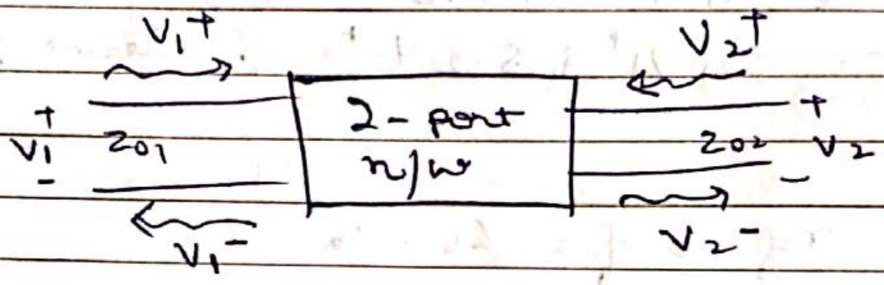
→ The second matrix is called scattering matrix.

→ The diagonal elements are called reflection co-efficient

→ The off-diagonal elements are called transmission co-efficients.

$$\text{ref co-eff} = \frac{\text{ref volt wave}}{\text{Inci volt wave}}$$

For Two-port n/w :→



$$S_{11} = \frac{b_1}{a_1} \qquad S_{12} = \frac{b_1}{a_2}$$

$$S_{21} = \frac{b_2}{a_1} \qquad S_{22} = \frac{b_2}{a_2}$$

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

In terms of voltage

$$S_{11} = \frac{V_1^-}{\sqrt{Z_{01}}} \Big/ \frac{V_1^+}{\sqrt{Z_{01}}} = \frac{V_1^-}{V_1^+}$$

$$S_{12} = \frac{V_1^-}{\sqrt{Z_{01}}} \Big/ \frac{V_2^+}{\sqrt{Z_{02}}} = \frac{V_1^-}{V_2^+}\sqrt{\frac{Z_{02}}{Z_{01}}}$$

$$S_{21} = \frac{V_2^-}{\sqrt{Z_{02}}} \Big/ \frac{V_1^+}{\sqrt{Z_{01}}} = \frac{V_2^-}{V_1^+}\sqrt{\frac{Z_{01}}{Z_{02}}}$$

$$S_{22} = V_2^-/V_2^+$$

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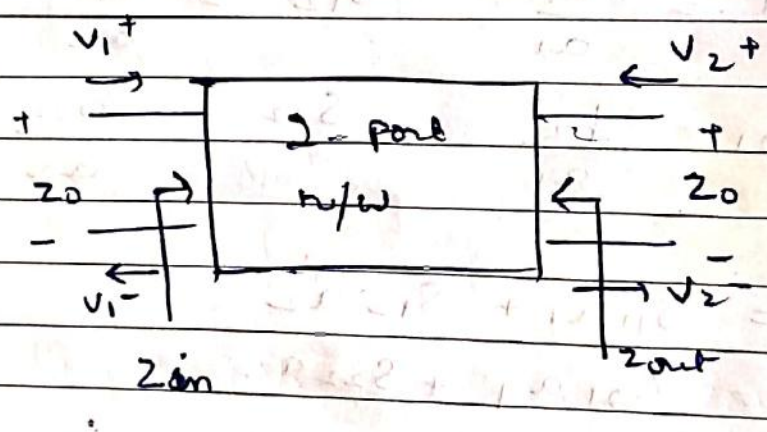
If $Z_{01} = Z_{02} = Z_0$ i.e; characteristic imp of both the ports is equal.

$$S_{11} = \frac{V_1^-}{V_1^+}, \quad S_{12} = \frac{V_1^-}{V_2^+}$$

$$S_{21} = \frac{V_2^-}{V_1^+}, \quad S_{22} = \frac{V_2^-}{V_2^+}$$

$$\left.\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ \end{aligned}\right\} \text{equations in terms of voltages.}$$

$$\text{ref. co-ef} = \frac{Z_L - Z_0}{Z_L + Z_0}$$



$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \qquad S_{22} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}$$

Properties of S-matrix :→

i) S-matrix is a square matrix of order $m \times m$ where m = no. of ports.

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ii, A n/w is referred to as reciprocal if $[S] = [S]^T$

iii, A n/w is said to be lossless if

$$[S]^T [S]^* = I$$

↓ Transpose ↓ Complex conjugate ↓ Identity matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$S^T = \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix}$$

$$S^* = \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix}$$

For lossless n/w :

$$\begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_{11}S_{11}^* + S_{21}S_{21}^* & S_{11}S_{12}^* + S_{21}S_{22}^* \\ S_{12}S_{11}^* + S_{22}S_{21}^* & S_{12}S_{12}^* + S_{22}S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_{11}S_{11}^* + S_{21}S_{21}^* = 1$$

$\Rightarrow |S_{11}|^2 + |S_{21}|^2 = 1$ $\Rightarrow$ $|S_{12}|^2 + |S_{22}|^2 = 1$

$\Rightarrow S_{11}S_{12}^* + S_{21}S_{22}^* = 0$

$\Rightarrow S_{12}S_{11}^* + S_{22}S_{21}^* = 0$

For an n-port n/w the condition for lossless network is the sum of sq. of magnitudes of each column of the S-matrix should be 1 e.g.)

$$S_3 = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}_{3\times 3}$$

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$$

& $|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1$

& $|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1$

→ The waves are related to voltages & currents at k^{th} port by the following relation

$$a_k = \frac{V_k^+}{\sqrt{Z_{0k}}} = I_k^+ \sqrt{Z_{0k}}$$

$$b_k = \frac{V_k^-}{\sqrt{Z_{0k}}} = I_k^- \sqrt{Z_{0k}}$$

where, Z_{0k} is the charac. imp. of the k^{th} port

Since, the voltage & current in tr line can be ~~presented~~ given by

$$V_k = V_k^+ + V_k^- = (a_k + b_k)\sqrt{Z_{0k}}$$

$$I_k = I_k^+ - I_k^- = \frac{a_k - b_k}{\sqrt{Z_{0k}}}$$

Thus, $a_k = \frac{1}{2}\left\{\frac{V_k^+ + V_k^-}{\sqrt{Z_{0k}}} + (I_k^+ - I_k^-)\sqrt{Z_{0k}}\right\}$

or $a_k = \frac{1}{2}\left\{\frac{V_k}{\sqrt{Z_{0k}}} + I_k\sqrt{Z_{0k}}\right\}$

||ly $b_k = \frac{1}{2}\left\{\frac{V_k}{\sqrt{Z_{0k}}} - I_k\sqrt{Z_{0k}}\right\}$

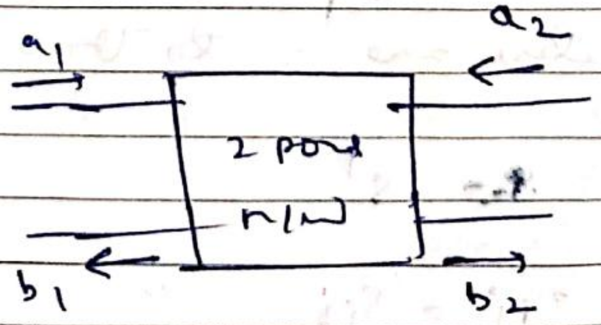
→ The diagonal elements of a scat. matrix are called ref. co-eff.

$$S_{kk} = \frac{b_k}{a_k} = \frac{V_k^-}{V_k^+} = \frac{I_k^-}{I_k^+}$$

→ The off diagonal elements of the ~~Tx co-ef~~ scat. matrix represent Tx. coefficients.

$$S_{ji} = \frac{b_j}{a_i} = \frac{V_j^-}{V_i^+}\sqrt{\frac{Z_{0i}}{Z_{0j}}} = \frac{I_j^-}{I_j^+}\sqrt{\frac{Z_{0j}}{Z_{0i}}}$$

Losses in n/w :→ In uw n/w its essential to represent losses in terms of S-parameters when the ports are terminated with matched loads.



a) Insertion loss :→ $\frac{P_{2}}{P}$

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$$IL(dB) = 10\log \frac{P_i}{P_d}$$

b) Transmission loss :→ $\frac{P_i - P_r}{P_o}$

$$T_L(dB) = 10\log \frac{P_i - P_r}{P_o}$$

c) Reflection loss :→ $\frac{P_i}{P_i - P_r}$

$$R_L(dB) = 10\log\left(\frac{P_i}{P_i - P_r}\right)$$

d) Return loss :→ $\frac{P_i}{P_r}$

$$ReL(dB) = 10\log \frac{P_i}{P_r}$$

Important properties of S-parameters :→

i, Symmetry Property :→ It states if a mw junc satisfies reciprocity condition, the junc is linear passive ckt. & then the S-parameters are = to their corresponding transpose

$$S = S^T$$

$$S_{ij} = S_{ji}$$

ii, Unity Property :→ It states that the sum of product of each terms in any row or column of S-matrix is multiplied by their complex conjugate, the result is unity.

$$\sum_{i=1}^{K} S_{ij} S_{ij}^{*} = 1 \qquad \text{for } j = 1, 2, \cdots n$$

iii, <u>Zero property</u> :→ The zero property states that the sum of product of each term in any row or column is multiplied by the complex conjugate of any other row or column is zero

$$\sum_{i=1}^{K} S_{ij} S_{ij}^{*} = 0 \qquad \text{for } j = 1, 2, 3 \cdots n$$

iv, <u>Phase Shift prop</u> :→ If any of the terminal planes (K^{th} port) is moved away from a junc by an electric distance $\beta_K L_K$ then each co-efficient of Sc. matrix of that port will be multiplied by a factor $e^{-j\beta_K L_K}$

$$S' = \theta S$$

$$\theta = \begin{bmatrix} \theta_{11} & 0 & 0 \\ 0 & \theta_{22} & 0 \\ \vdots & \ddots & \\ 0 & \cdots & \theta_{nn} \end{bmatrix}$$

where $\theta_{11} = \theta_{22} = \theta_{nn} = e^{-j\beta_K L_K}$

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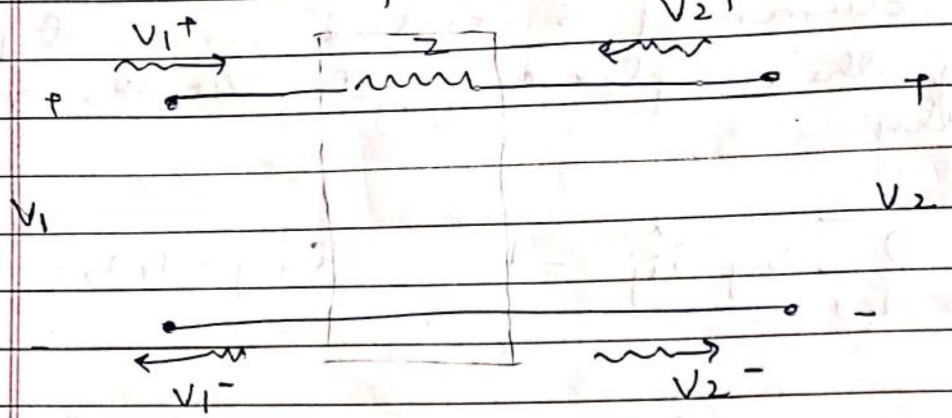
Example of S-parameter :→

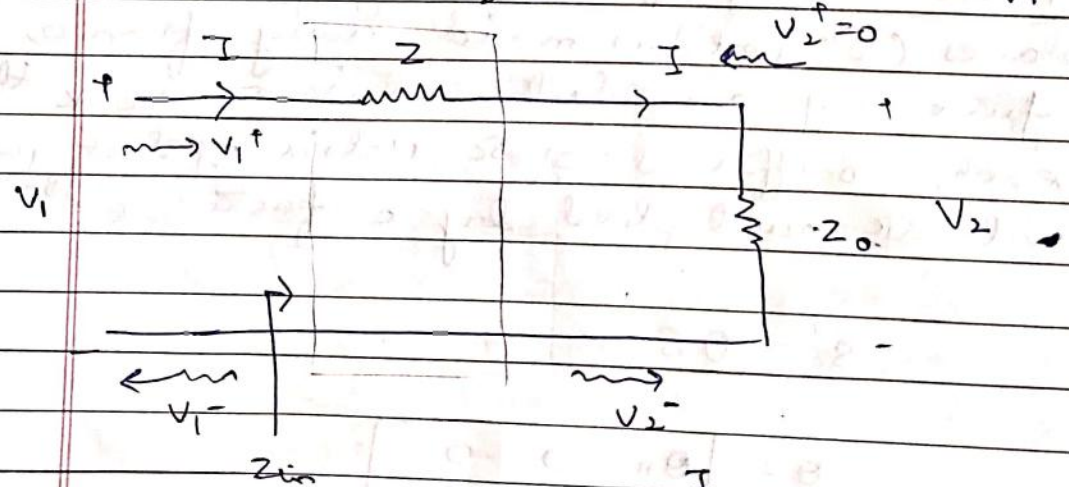
↳ S- matrix of series element



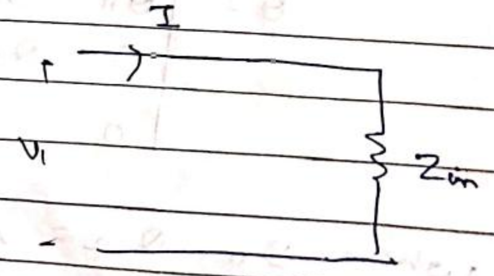
→ There will be zero reflection when the port is perfectly matched = to char. imp

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$
$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} , \quad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$



① — $Z_{in} = Z + Z_0$



② — $V_1 = I Z_{in} = I(Z + Z_0)$

ref co-ef

$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z + Z_0 - Z_0}{Z + Z_0 + Z_0}$$

$$\boxed{S_{11} = \frac{Z}{Z + 2Z_0}}$$

$$V_2 = V_2^+ + V_2^-$$

$$V_2 = V_2^-$$

$$V_2 = I Z_0$$

$$V_2^- = I Z_0$$

$$V_2^- = \frac{V_1}{(Z+Z_0)} Z_0$$

$$V_2^- \left(\frac{Z+Z_0}{Z_0} \right) = V_1 = V_1^+ + V_1^-$$

÷ing b-s by V_1^+

$$\frac{V_2^-}{V_1^+} \left(\frac{Z+Z_0}{Z_0} \right) = \frac{V_1^+}{V_1^+} + \frac{V_1^-}{V_1^+}$$

$$S_{21} \left(\frac{Z+Z_0}{Z_0} \right) = 1 + S_{11}$$

$$S_{21} = \left(\frac{Z_0}{Z+Z_0} \right) \times \left(1 + \frac{Z}{Z+2Z_0} \right)$$

$$= \left(\frac{Z_0}{Z+Z_0} \right) \left(\frac{Z + 2Z_0 + Z}{Z + 2Z_0} \right)$$

$$S_{21} = \left(Z_0 \right) 2 \left(Z + \cancel{Z_0} \right) = \frac{2Z_0}{}$$

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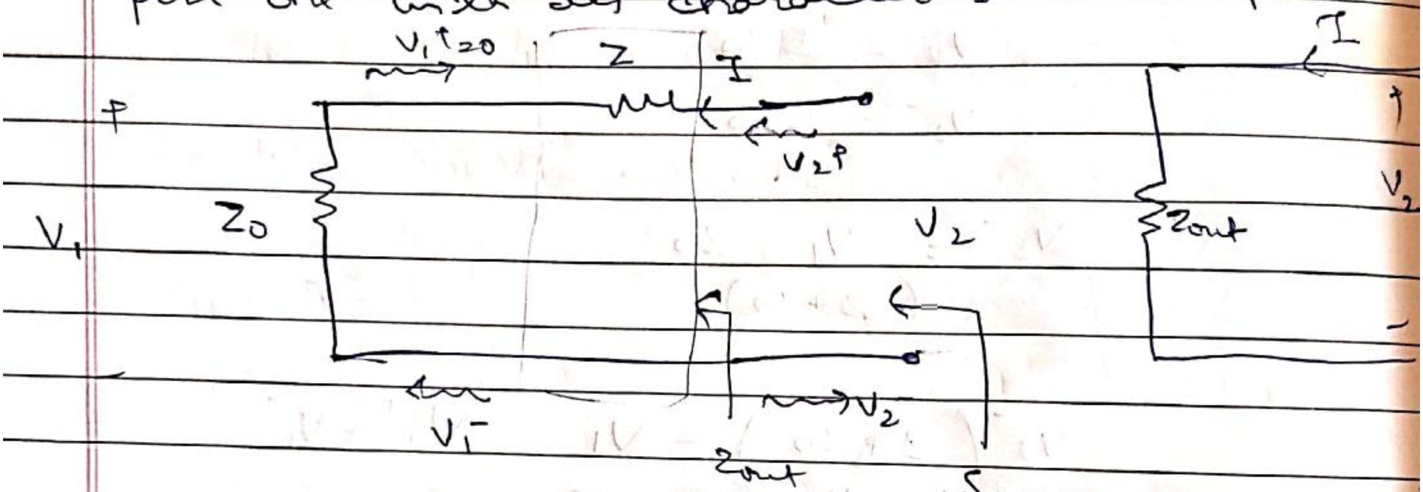
$$\boxed{S_{21} = \frac{2Z_0}{Z+2Z_0}}$$

$$S_{12} = \frac{V_1^-}{V_2^+}\Bigg|_{V_1^+=0} \qquad S_{22} = \frac{V_2^-}{V_2^+}\Bigg|_{V_1^+=0}$$

for making $V_1^+=0$ we have to terminate port one with its characteristic imp.



$S_{22} =$

$$Z_{out} = Z + Z_0 \quad \text{—— (8)}$$

$$S_{22} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} = \frac{Z + Z_0 - Z_0}{Z + Z_0 + Z_0}$$

$$\boxed{S_{22} = \frac{Z}{Z+2Z_0}}$$

$$V_2 = I Z_{out}$$

$$V_2 = V_2^+ + V_2^- = I Z_{out}$$

$$V_1 = I Z_0$$

$$V_1^+ + V_1^- = I Z_0$$

$$V_1^- = I Z_0 \quad \text{or} \quad I = V_1^-$$

$$V_2^+ + V_2^- = \frac{V_1^-}{Z_0} Z_{out} = \frac{V_1^-}{Z_0}(Z+Z_0)$$

$\div$ ing b.s by V_2^+

$$1 + \frac{V_2^-}{V_2^+} = \frac{V_1^-}{V_2^+}\left(\frac{Z+Z_0}{Z_0}\right)$$

$$1 + S_{22} = S_{12}\left(\frac{Z+Z_0}{Z_0}\right)$$

$$\left(1 + \frac{Z}{Z+2Z_0}\right) = S_{12}\left(\frac{Z+Z_0}{Z_0}\right)$$

$$\left(\frac{Z+2Z_0+Z}{Z+2Z_0}\right) = S_{12}\left(\frac{Z+Z_0}{Z_0}\right)$$

$$2\left(\frac{\cancel{Z+Z_0}}{Z+2Z_0}\right) = S_{12}\left(\frac{\cancel{Z+Z_0}}{Z_0}\right)$$

$$\boxed{S_{12} = \frac{2Z_0}{Z+2Z_0}}$$

$$S = \begin{bmatrix} \frac{Z}{Z+2Z_0} & \frac{2Z_0}{Z+2Z_0} \\ \frac{2Z_0}{Z+2Z_0} & \frac{Z}{Z+2Z_0} \end{bmatrix}$$

$[S]^T = [S]$ showing its reciprocal n/w.

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Microwave Hybrid Circuits :→ A µ wave ckt consists of several µ wave devices connected in some way to achieve desired transmission of µ wave signals. The interconnection of µ wave devices may be regarded as a µ wave junc. e.g;

i, E-plane Tee ii, H-plane Tee
iii, Magic tees iv Hybrid ring
v, Directional coupler vi circulator

Tee Junction :→ In µW ckts a waveguide or coaxial line junc. with 3-independent ports is called a tee junc. From S parameter theory µω. Tee junc. should be characterised by a matrix of 3rd order & 9-elements. The characteristics of a 3-port junc can be explained by 3- theorems for tee junc.

i, Short ckt may always be placed in one of the arms
ii, If the junc is symmetric abt. one of its arms, a s/c can be placed in that arm.
iii, It is impossible for a 3-port junc. to present matched impedances at all three arms.

E-plane tee (Series Tee) :→ An E-plane tee is a waveguide tee in which the axis of side arm is ∥ to E field of the main guide. If collinear arms are symmetric about side arm, there are 2-diff trans characteristics.

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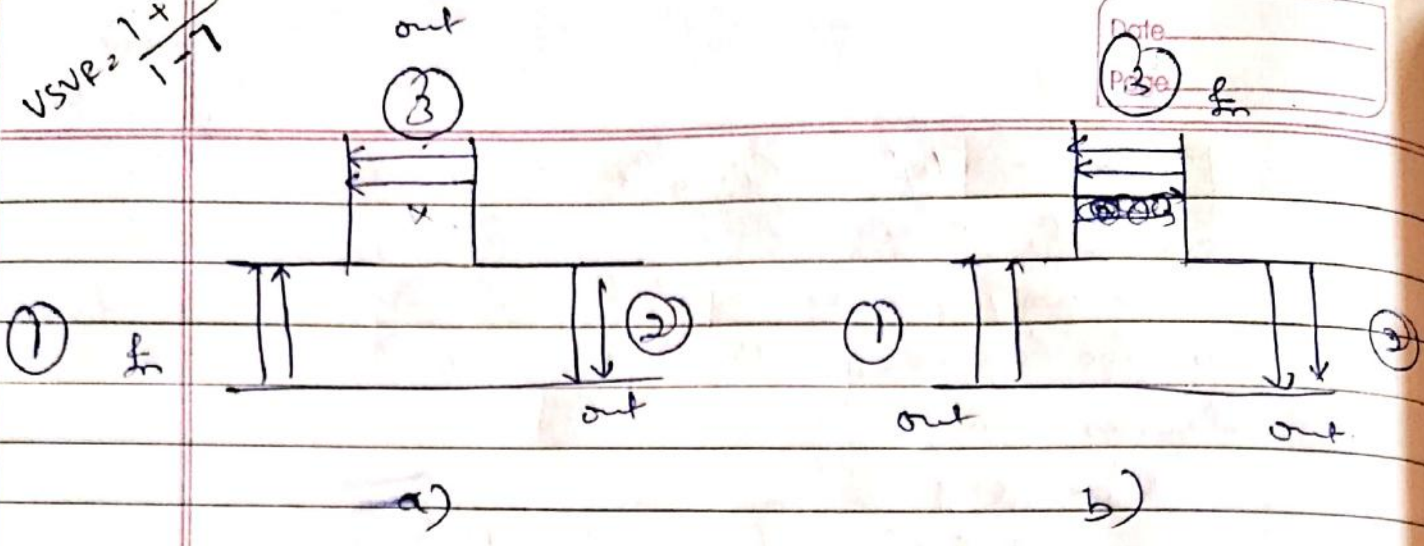
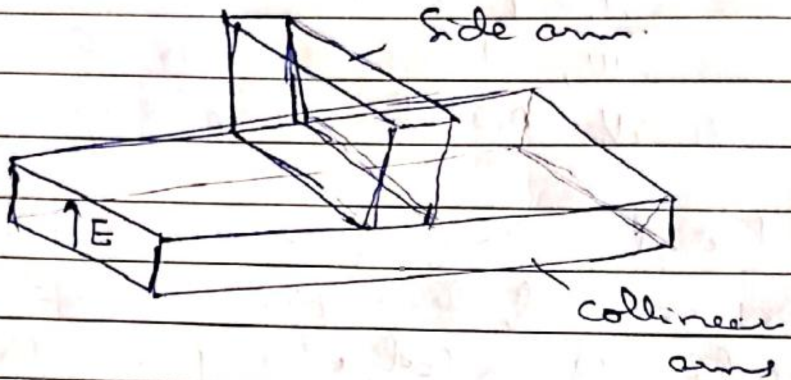
$VSWR = \frac{1+\Gamma}{1-\Gamma}$





→ If all the three ports are perfectly matched then

$$S_{11} = S_{22} = S_{33} = 0$$

→ If waves are fed to side arm 3 the waves at port 1 & 2 are equal in mag & opp in phase

$$S_{13} = -S_{23}$$

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

→ from symmetry prop.

$$S_{12} = S_{21}, \quad S_{13} = S_{31}, \quad S_{23} = S_{32}$$

→ From zero prop.

$$S_{31} \cdot S_{32}^{*} = 0$$

$$\text{or } S_{31} \cdot S_{23}^{*} = 0$$

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i.e; either S_{13} or S_{23}^* or both $= 0$

→ According to unity prop.

$$S_{21} \cdot S_{21}^* + S_{31} \times S_{31}^* = 1 \quad — (1)$$

$$S_{12} \cdot S_{12}^* + S_{32} \cdot S_{32}^* = 1 \quad — (2)$$

$$S_{13} \cdot S_{13}^* + S_{23} \cdot S_{23}^* = 1 \quad — (3)$$

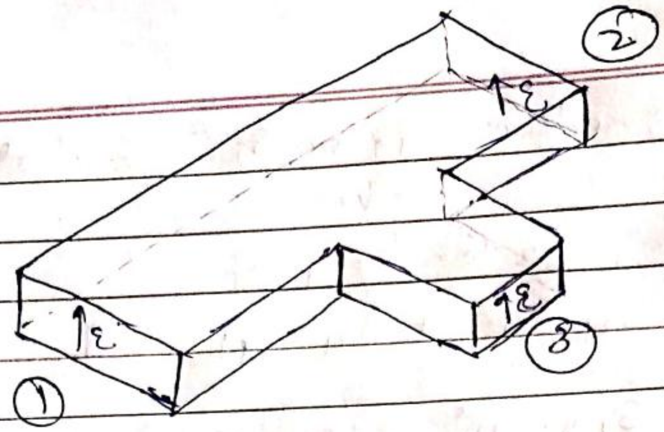
$$|S_{12}|^2 = 1 - |S_{13}|^2 = 1 - |S_{23}|^2 \quad — (4)$$

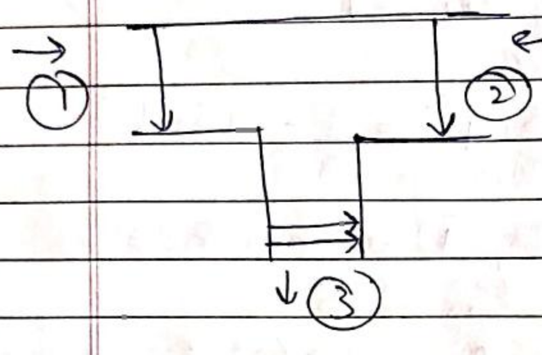
If $S_{13} = 0$ eq (4) is not holding as long as S_{23} is also $= 0$
If both are 0 then eq (3) is incorrect
therefore all 3 ports of this tee can't be perfectly matched

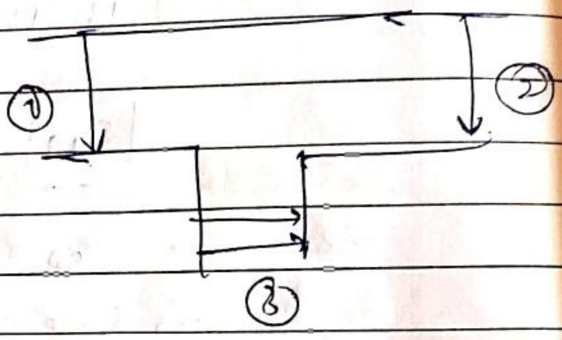
→ Since collinear arm is symmetric about side arm then $|S_{13}| = -|S_{23}|$ & $S_{11} = S_{22}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix}$$

H-plane tee (Shunt tee) :→ An H-plane tee is a waveguide tee in which the axis of its aside arm is shunting 'E' field or parallel to H field of main waveguide. Power fed at port 3 gets equally divided b/w port (1) & port (2) & is in phase





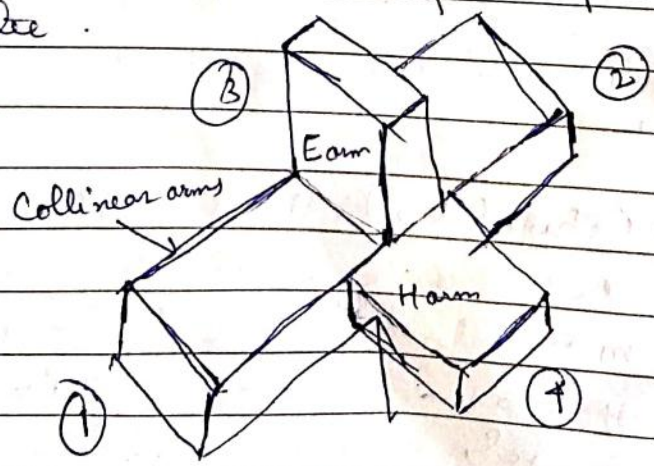


$\Longrightarrow$ $S_{13} = S_{23}$ in H plane

Unlike $S_{23} = -S_{13}$ in E plane.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix}$$

<u>Magic tee (Hybrid tee)</u> :→ A magic tee is a combination of E-plane tee & H-plane tee.



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1. If 2 waves of equal mag & same phase are fed to port 1 & 2, the o/p will be '0' at port 3 & additive at port 4

2. If a wave is fed to port 4, it will be equally divided b/w port 1 & port 2 & 0 will appear at port 3.

3. If a wave is fed to port 3, it will produce an o/p of equal mag but opp. phase at port 1 & port 2. O/P at port 4 will be 0 i.e; $S_{43} = S_{34} = 0$

4. If a wave is fed into one of the collinear arms at port ① or port 2 it will not appear at other collinear arm i.e; $S_{12} = S_{21} = 0$

$$\therefore S = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

They are commonly used for:
i) Mixing
ii) Duplexing
iii) Impedance measurements

Application :-
Suppose we have 2 identical radar Txs. A particular application requires twice

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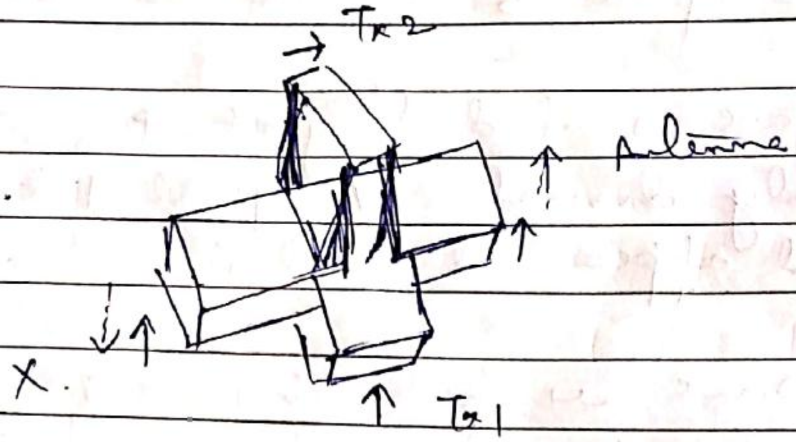
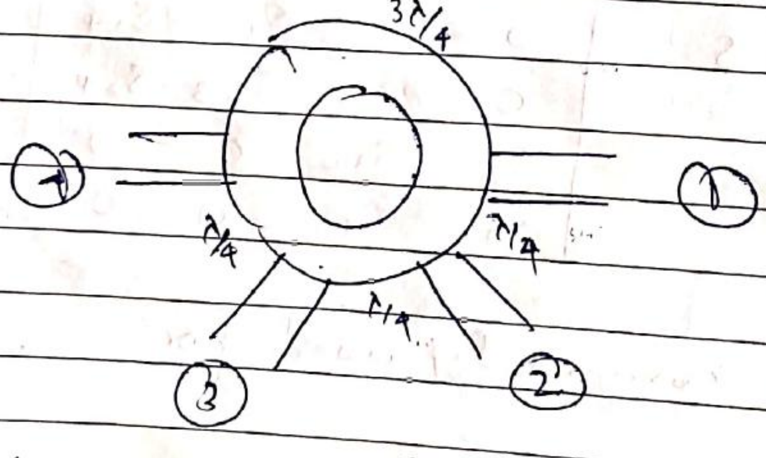
more input power to an antenna than either Tx can deliver



Hybrid Ring (Rat Race Junc/Ckt)

A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which 4 arms are connected at proper intervals by means of series or parallel junctions.

It has characteristics similar to ~~those~~ of hybrid tee. When a wave is fed at



at port ① will not appear at port ③ for the difference of phase shifts for waves travelling in clockwise & counterclockwise directions. Ily ~~the~~ the wave fed at ② will not appear at port ④

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Waveguide Corners, Bends & Twist :→

These components are usually used to change the direction of the guide through an arbitrary angle. In order to minimize reflections from discontinuities, it's desirable to have mean length b/w discontinuities = odd number of quarter wavelength

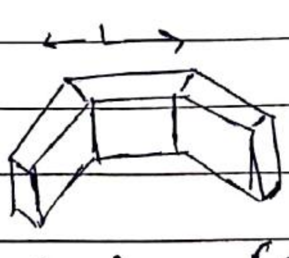
$$L = (2n+1)\frac{\lambda}{4}$$

If mean Length L follows above conditions the reflected waves from both the ends cancel out.

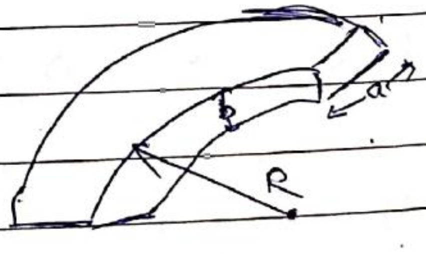
For wave guide bend

$R = 1.5a$ for H bend.

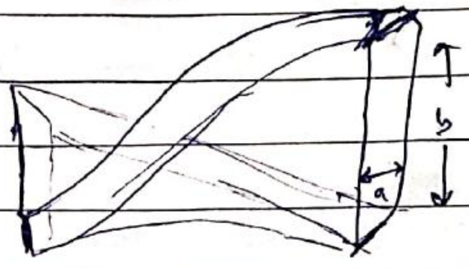
$R = 1.5b$ for E-bend.



E plane corner

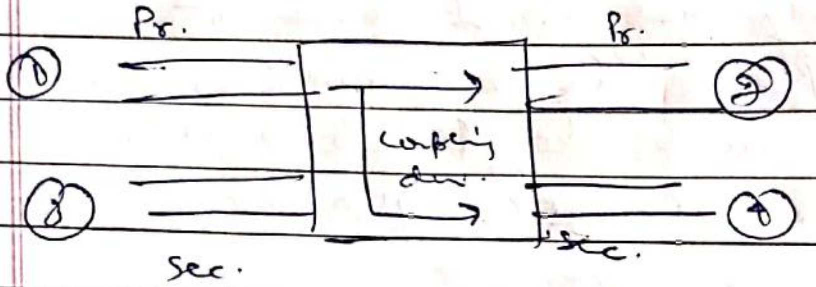


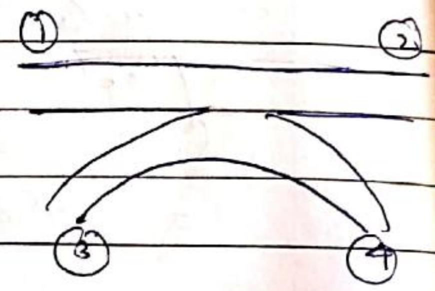
Bend



Twist.

Directional Couplers:→ It's a 4-port waveguide junc. It consists of a primary & a secondary waveguide. When all ports are terminated with ch. Impedances, there is free transmission of power b/w port ① & ② & no transmission b/w port ① & ③ or b/w port ② & ④ because no coupling exists b/w these two pairs of ports.





$$\text{coupling factor (dB)} = 10 \log \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log \frac{P_4}{P_3}$$

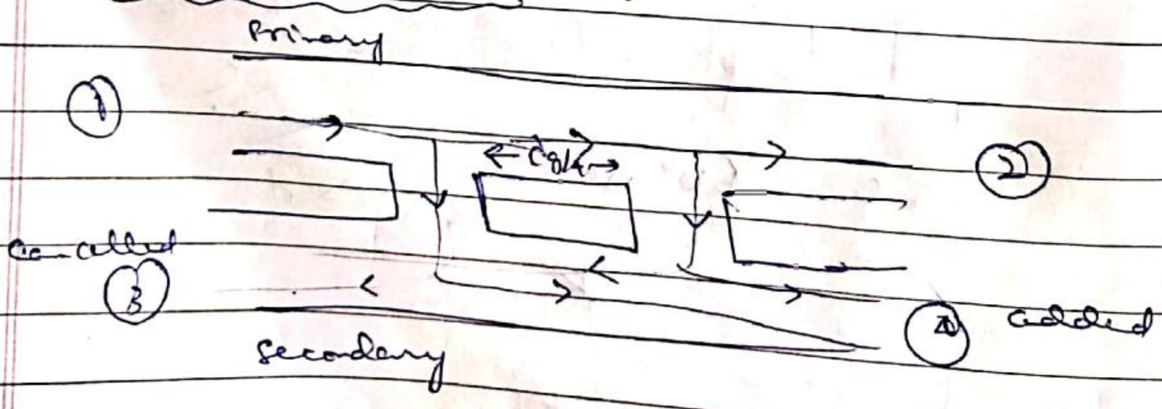
$$\text{Isolation (dB)} = 10 \log \frac{P_1}{P_3}$$

Types of directional couplers:

i, 2-hole ii, 4-hole iii, Reverse coupling

iv, Bethe hole

Two hole D.C :→



Forward waves in Sec. are in phase and are added at ④ Backward waves are out of phase and are cancelled

S- Matrix of a directional coupler where all 4- ports are completely matched. The diagonal elements are

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

No - coupling b/w port 1 & 3 & between port 2 & 4. So, $S_{13} = S_{31} = S_{24} = S_{42} = 0$

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

→ From zero property of S- matrix

$$S_{12} S_{14}^* + S_{32} \cdot S_{34}^* = 0 \quad \text{--- (1)}$$

$$S_{21} S_{23}^* + S_{41} \cdot S_{43}^* = 0 \quad \text{--- (2)}$$

→ from unity property

$$S_{12} S_{12}^* + S_{14} \cdot S_{14}^* = 1 \quad \text{--- (3)}$$

$|S_{21}|^2 + |S_{41}|^2 = 1$

$|S_{12}|^2 + |S_{32}|^2 = 1$

$|S_{23}|^2 + |S_{43}|^2 = 1$

$|S_{14}|^2 + |S_{34}|^2 = 1$

From (1) & (2)

$$|S_{12}| \, |S_{14}| = |S_{32}| \cdot |S_{34}|$$

$$|S_{21}| \, |S_{23}| = |S_{41}| \cdot |S_{43}|$$

$S_{12} = S_{34}$

Let $S_{12} = S_{34} = p$ where p is positive & real

from eq (2)

$p(S_{23}^{*} + S_{41}) = 0$

Let $S_{23} = S_{41} = jq$

where q is +ve & real

$p^2 + q^2 = 1$ — from (3)

$$S = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

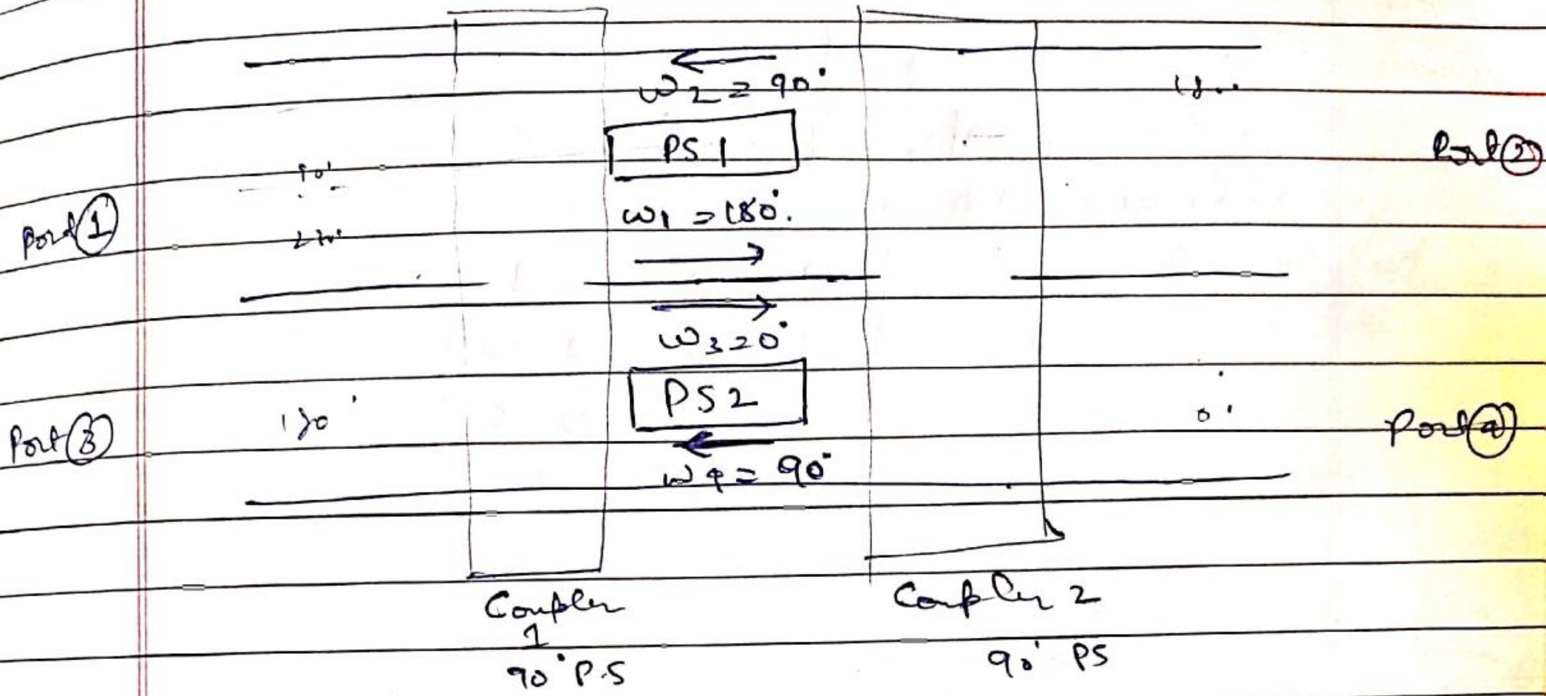
Circulator & Isolator :→

Both µwave circulators & µwave isolators are non reciprocal transmission devices that use the property of faraday rotation.

Micro wave Circulator :→ A µwave Circulator is a multi port waveguide junction in which the wave can flow only from nth port to (n+1)th port in one direction. There can be any number of ports but the mostly commonly used circulator is with 4 ports.

Scanned with CamScanner

One type of 4-port μ wave Circulator is a combination of 2 3-dB side hole directional couplers, a rect waveguide & 2 non-reciprocal phase shifters



→ when a wave is incident at port (1) it reaches (2) at a phase of 180° from two different ends / holes. It reaches port (4) at a phase of 90° & 270° i.e) 180° out of phase. So, the wave incident at port 1 is recieved at port (2).

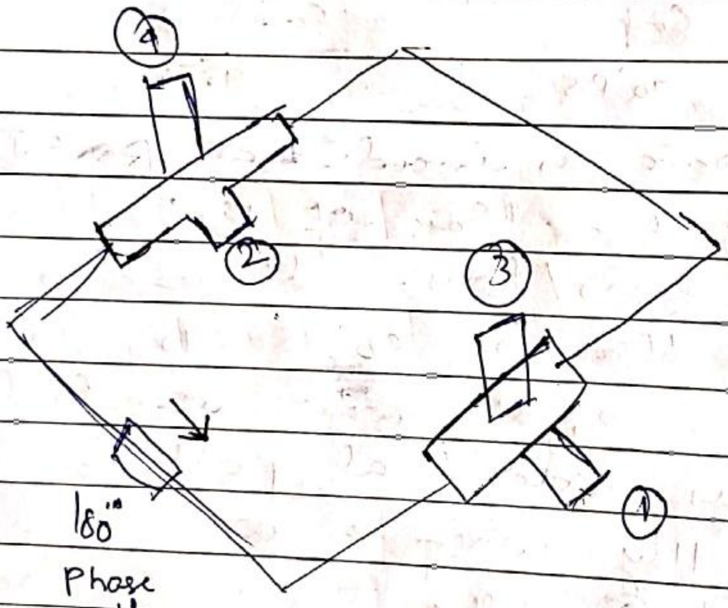
Illy. when a wave is incident from port (2) the two waves reaching port (1) is 270 & 90° at phase so, they cancel out. The waves at port (3) are both at a phase of 270°, so they add up.

→ A perfectly matched, lossless, & non-reciprocal 4-port Circulator has an S matrix of the form:

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

Using the properties of S-matrix (Unity & Zero property) ∴ S-matrix can be reduced to

$$S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

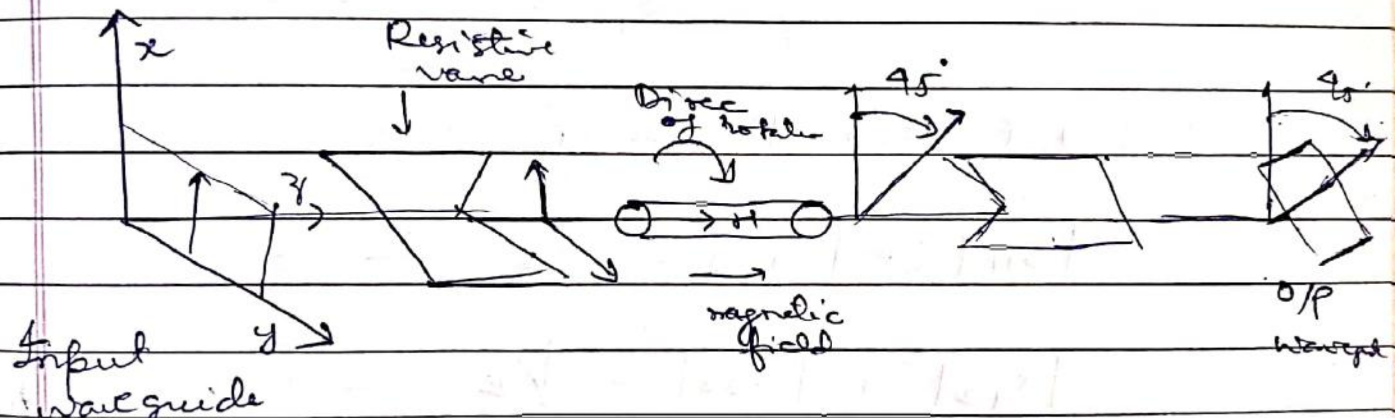


180° Phase shift (non-reciprocal)

1-2
2-3
3-4
4-1

Micro wave Isolators :→ An isolator is a non reciprocal transmission device that is used to isolate ~~of~~ one component from reflections of other components in the transmission line. An ideal isolator completly absorbs ~~reflection of other~~ the power for propogation in one direction

and provides lossless transmission in the opposite direction. Thus the isolator is usually called uniline. Isolators are used to improve the freq. stability of µwave generators like klystrons & magnetrons in which the reflection from the load affects the generating freq.
Isolators can be made in many ways like terminating port ③ & ④ of a 4 port Circulator with matched loads. The other way is by inserting ferrite rod in a rectangular waveguide



→ E plane Tee S-matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

① It's a reciprocal n/w so,

$$S_{12} = S_{21} \ \& \ S_{13} = S_{31} \ ; \ S_{23} = S_{32}$$

② ~~Since port ③ is perfectly matched~~

$$S_{33} = 0$$

③ Since power I/P at 3 is equal in mag & opp in phase so $S_{13} = - S_{23}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

Using Unitary property of S-matrix

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{— (A)}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{— (B)}$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{— (C)}$$

From (C)

$$2|S_{13}|^2 = 1$$

$$S_{13} = \frac{1}{\sqrt{2}} \quad \text{— (D)}$$

Substituting D in (A) & (B)

$$|S_{11}|^2 + |S_{12}|^2 = 1/2 \quad \text{— (E)}$$

$$|S_{12}|^2 + |S_{22}|^2 = 1/2 \quad \text{— (F)}$$

Comparing (E) & (F)

$$\boxed{S_{11} = S_{22}}$$

Using Zero property we have.

$$S_{11} S_{12}^* + S_{12} S_{22}^* - S_{13} S_{13}^* = 0$$

$$2(S_{11} S_{12}^*) - \frac{1}{2} = 0$$

$$(S_{11} S_{12}^*) = \frac{1}{4} \quad \text{— (G)}$$

Also $S_{11} S_{13}^* - S_{12} S_{13}^* = 0$

$$S_{11} \frac{1}{\sqrt{2}} - S_{12} \frac{1}{\sqrt{2}} = 0$$

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$$\frac{1}{\sqrt{2}}(S_{11} - S_{12}) = 0$$

$$S_{11} = S_{12} \quad \text{— (H)}$$

Substituting in (G)

$$S_{11} = S_{12} = S_{22} = \frac{1}{2}.$$

$$S = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}.$$

For H-plane tee :→

(1) Port 3 (H-arm) is perfectly matched

i.e; $S_{33} = 0$

(2) O/P at port (1) & (2) is equal & in phase if I/P is given at port (3)

i.e, $S_{13} = S_{23}$

(3) S-matrix for this symmetric

$S_{12} = S_{21}$, $S_{23} = S_{32}$, $S_{13} = S_{31}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

As per Identity:

$$[S][S]^* = I$$

~~or we can say it~~ follows Unitary prop
~~and zero prop~~

Unitary prop:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{— (A)}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{— (B)}$$

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$2|S_{13}|^2 = 1$$

From (A), (B)

$$S_{13} = \frac{1}{\sqrt{2}} \quad \text{— (C)}$$

$$\boxed{S_{11} = S_{22}} \quad \text{— (D)}$$

Zero prop.

$$S_{11} S_{12}^* + S_{12} S_{22}^* + S_{13} S_{23}^* = 0 \quad \text{— (E)}$$

$$S_{11} S_{13}^* + S_{12} S_{23}^* = 0 \quad \text{— (F)}$$

~~Using (D) in (E)~~

$$\cancel{2 S_{11} S_{12}^* + \frac{1}{2} = 0}$$

$$\cancel{S_{11} S_{12}^* = -\frac{1}{4}}$$

From (F) $S_{13}^*(S_{11} + S_{12}) = 0$

~~So~~

$$-|S_{12}|^2 \cancel{+} - |S_{22}|^2 = -\frac{1}{2}$$

$$\cancel{-}2 S_{22}^2 = \cancel{-}\frac{1}{2}$$

$$S_{22} = \frac{1}{2}$$

$$S_{11} = S_{22} = \frac{1}{2}$$

$$S_{12} = -\frac{1}{2}$$

$$S = \begin{bmatrix} \frac{1}{2} & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

→ Magic Tee :→

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

→ $S_{33} = 0$, $S_{44} = 0$ perfectly matched

→ $S_{34} = S_{34} = 0$ Isolated ports

→ Reciprocal i.e; $S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{14} = S_{41}$, $S_{23} = S_{32}$, $S_{24} = S_{42}$ & $S_{34} = S_{43}$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & 0 \end{bmatrix}$$

→ Unitary property

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{— (1)}$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{23}|^2 + |S_{24}|^2 = 1 \quad \text{— (2)}$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \quad \text{— (3)}$$

$$|S_{14}|^2 + |S_{24}|^2 = 1 \quad \text{— (4)}$$

We know,

$S_{23} = -S_{13}$, $S_{24} = S_{14}$

For E arm   For H-arm

Substituting in (3) & (4)

$$S_{24} = S_{23} = S_{14} = \frac{1}{\sqrt{2}}$$

$$S_{13} = -1/\sqrt{2}$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$|S_{12}|^2 + |S_{22}|^2 = 0$$

$$\boxed{S_{11} = S_{22}}$$

→ Zero prop

$$S_{11} S_{13}^* + S_{12} S_{23}^* = 0 \quad \text{— (5)}$$

$$S_{11} S_{14}^* + S_{12} S_{24}^* = 0 \quad \text{— (6)}$$

$$S_{11} = -S_{12}$$

(from 6)

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$$-S_{12}\left(-\frac{1}{\sqrt{2}}\right) + S_{12}\frac{1}{\sqrt{2}} = 0 \quad \because \text{from } ⑤$$

$$\cancel{\frac{S_{12}}{\sqrt{2}}} \;\cancel{= -\frac{S_{12}}{\sqrt{2}}}$$

$$\frac{2\,S_{12}}{\sqrt{2}} = 0$$

$$S_{12} = 0$$

$$S_{12} = S_{11} = S_{22} = 0$$

$$[S] = \begin{bmatrix} 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

~~S_{12}~~

$$S_{11}S_{13}^* + S_{12}S_{23}^* = 0$$

$$S_{11}S_{14}^* + S_{12}S_{24}^* = 0$$

$$S_{11}\frac{1}{\sqrt{2}} + S_{12}\frac{1}{\sqrt{2}} = 0$$

$$S_{11} = -S_{12}.$$

$$S_{11}S_{14}^* - S_{11}S_{24}^* = 0$$

$$-S_{11}\frac{1}{\sqrt{2}} - S_{11}\frac{1}{\sqrt{2}} = 0$$

$$-2\frac{1}{\sqrt{2}}S_{11} = 0$$