LECTURE # 09 - Topic 03 (Part-II)

3.2.4.2 Double Side Band - Suppressed Carrier (DSB-SC)

As the name suggests, the DSB-SC signal has two side band components ie USB & LSB components but the carrier of DSB-Full carrier is suppressed. In other words we can say that the major difference of a DSB-Full carrier & DSB-suppressed carrier is the carrier component which is suppressed. This can be explained by comparing the illustrations of DSB-Full carrier & DSB-SC for both their Time domain & frequency domain Fig 1(a,b)

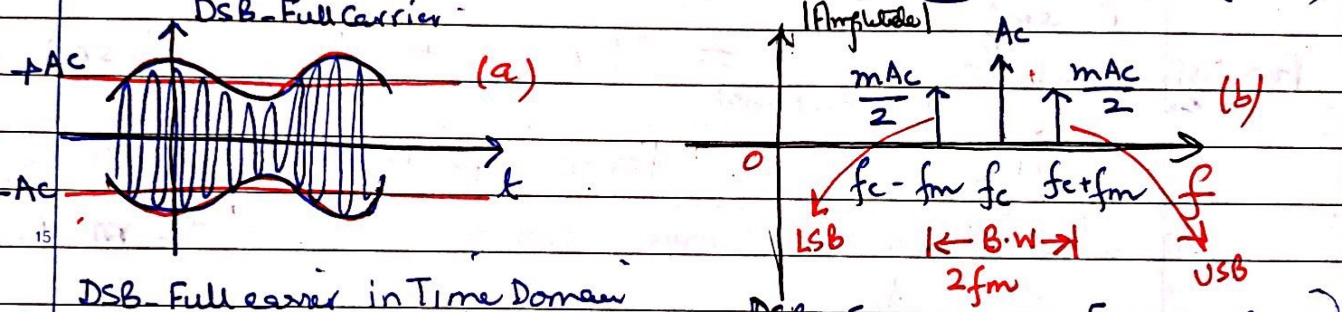


Fig 1(a,b): DSB-Full carrier in Time domain & Frequency Domain.

After suppression of carrier Fig 2(a,b) result.

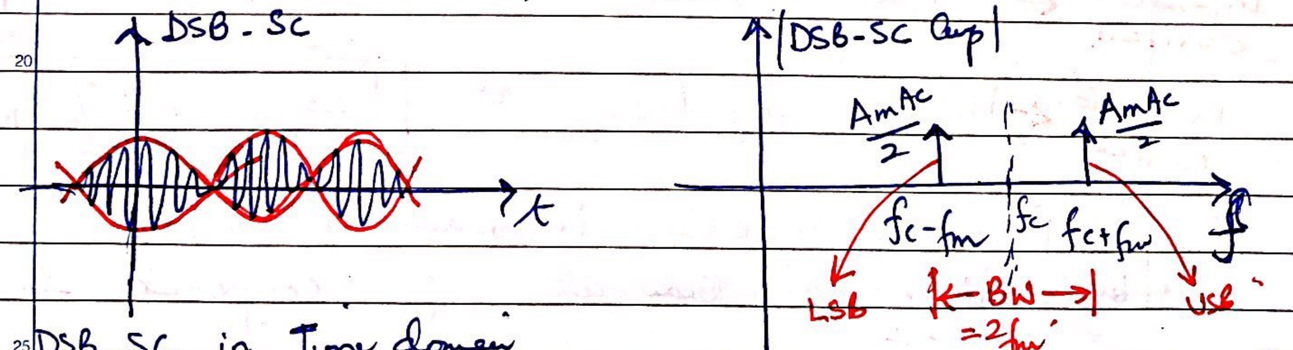


Fig 2(a,b) :- DSB-SC in Time Domain & in Frequency domain

Above figure shows the result when the carrier of DSB-Full carrier is suppressed. Now the spectrum of Fig 2(b) show USB & LSB but no f_c component. Fig 2(b) is same as a frequency translation of base band signal around f_c carrier position.

Q. Why is DSB-SC modulation scheme implemented? In other words why do we go for DSB-SC? And what are its Advantages & Disadvantages as compared to DSB-SC?

DSB-SC modulation scheme was introduced to overcome some drawback of DSB-Full carrier Scheme.
This drawback of DSB-Full carrier is associated with its Power relations that was explained on pg. 92 of Lecture # 08.
There it was explained that total Transmitted power

For DSB-Full carrier :- $P_{T(DSB-Full carrier)} = P_{carrier} + P_{LSB} + P_{USB}$.

$$\boxed{P_{Total} = P_{carrier} + P_{carrier}\left(\frac{m^2}{4}\right) + P_{carrier}\left(\frac{m^2}{4}\right)}$$

Then for maximum permissible value of $m=1$ we show

No information carried ← $P_{carrier} = \frac{2}{3} P_{Total}$ or $\boxed{P_{carrier} = 66.6\% \, P_{Total}}$

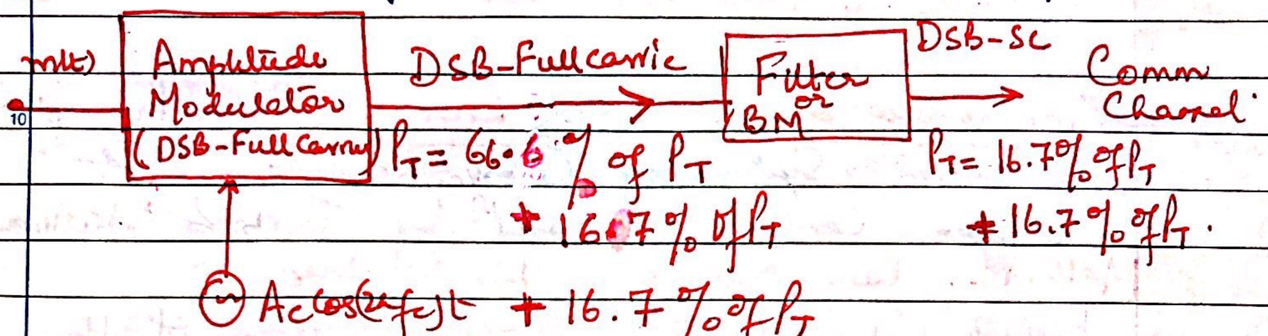
Information carried ← $P_{USB} = \frac{1}{6} P_{Total}$ or $\boxed{P_{USB} = 16.7\% \, P_{Total}}$

Information carried ← $P_{LSB} = \frac{1}{6} P_{Total}$ or $\boxed{P_{LSB} = 16.7\% \, P_{Total}}$

From the spectrum information is carried only by $mA_c/2$ amplitude at $f_c - f_m$ component & $mA_c/2$ amplitude at $f_c + f_m$ component only.
The utility of carrier is only to carry the message signal $m(t)$.
We can still recover and detect the base band message at Rx either from USB or LSB.
There is no requirement of carrier.
This detection at Rx is only possible by using a product detector which implements Reverse Frequency Translation. Envelope Detector cannot

be used. So if a product detector can be used at Rx to recover m(t) message from the USB & LSB only then there is no need to transmit Carrier (f_c component) in the first place. So the carrier (f_c component) might as well be cancelled. This will provide an advantage of saving 66.6% of $P_{transmitted}$

$P_T \rightarrow$ Total Power



Above illustration shows that if a filter is used or a Balanced Modulator (BM) is used to cancel carrier the actually it would cut 66.6% of P_T. This power could either be saved or the saved power could be re-utilized by LSB & USB to increase their transmitted power over much longer distances

Q. Is their any benefit of B.W conservation?

No. There is no benefit of B.W conservation in case of DSB-SC (See figure 2(b)). Only now the carrier component is suppressed and still max frequency component is f_m and here also $B.W = 2f_m$.

Thumb's rule 3.5. $P_{Total} = \underbrace{66.6\%(P_T)}_{\text{Total Power}} + \underbrace{16.6\%(P_T)}_{\text{Carrie Power}}$ (LSB-Power) $+ \underbrace{16.6\%(P_T)}_{\text{USB-power}}$

Thumb's rule 3.6. $(B.W)_{\text{DSB-Fullcarrier}} = (B.W)_{\text{DSB-SC}}$

Thumb rule 3.7 : Generation of DSB-Full carrier by Amplitude Modulator.

Generation of DSB-SC by Balanced Modulator (BM) or Carrier-Suppression Filter

Detection of DSB-Full carrier by Envelope Detector or Product Detector.

Detection of DSB-SC by Product detector and not by Envelope Detector.

Generation of DSB-SC Signal [Refer Principles of Communication Systems by Taub & Schilling & Electronic Communication System by George Kennedy]

Method 1: Using Balanced Modulator

Fig 3 below show the block diagram of the set-up used for generation of DSB_SC signal. This is called a Balanced Modulation or in short form BM. The Balanced Modulator is so called because it is a balanced configuration of two Amplitude Modulators. Thus the Amplitude Modulator is an essential building block of a BM for generation of DSB-FC.

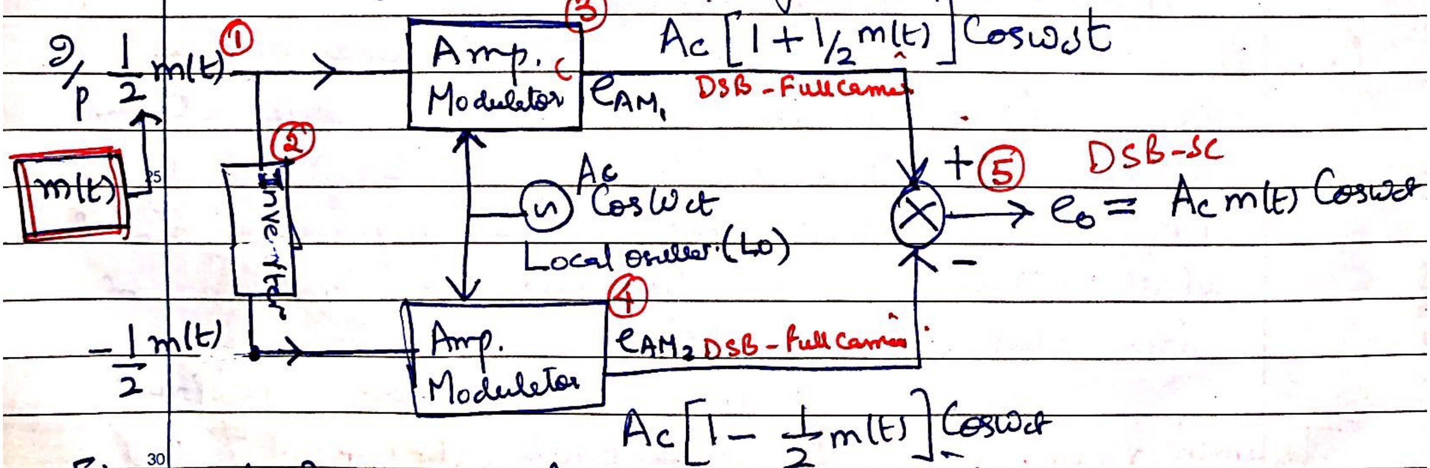


Fig 3: A Balanced Modulator configuration.

① Input at ① = message signal is $m(t)$. Half of it is passed through inverter.

② Output of inverter = This will be $-\frac{1}{2}m(t)$.

(27)

③ Amplitude Modulator o/p (e_{AM_1}) :- In accordance with Equation (2) on page 86 of Lecture #08 the output is

$$\boxed{e_{AM_1} = A_c\left[1 + \frac{1}{2}m(t)\right]\cos\omega_c t}.$$

Receiving the carrier $A_c\cos\omega_c t$ from LO

④ Amplitude Modulator o/p (e_{AM_2}) :- Similarly output

$$\boxed{e_{AM_2} = A_c\left[1 - \frac{1}{2}m(t)\right]\cos\omega_c t}.$$

This modulator also receives same carrier from same LO.

⑤ Final o/p $e_{o/p} = e_{AM_1} - e_{AM_2}$

$$e_{o/p} = A_c\left[1 + \frac{m(t)}{2}\right]\cos\omega_c t - A_c\left[1 - \frac{m(t)}{2}\right]\cos\omega_c t$$

$$\therefore e_{o/p} = \cancel{A_c\cos\omega_c t} + \frac{A_c m(t)\cos\omega_c t}{2} - \cancel{A_c\cos\omega_c t} + \frac{A_c m(t)\cos\omega_c t}{2}.$$

$$\therefore \boxed{e_{o/p} = A_c m(t)\cos\omega_c t} \leftarrow \text{DSB-SC}.$$

Q. Show that $e_{o/p} = A_c m(t)\cos\omega_c t$ is a DSB-SC signal

$$e_{o/p} = A_c m(t)\cos\omega_c t.$$

Let $m(t) = m\cos\omega_m t$

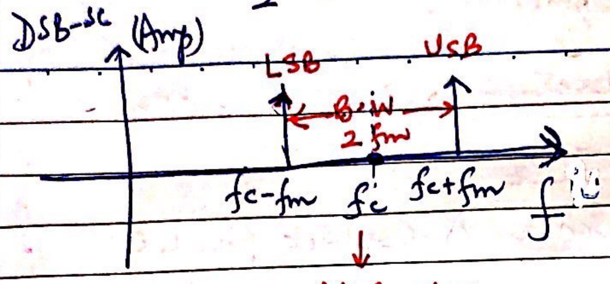
$$\left\{\begin{array}{l}\omega_c >> \omega_m \\ \omega_c = 2\pi f_c \\ \omega_m = 2\pi f_m\end{array}\right\}$$

$$\therefore e_{o/p} = A_c m\cos\omega_c t\cos\omega_m t$$

Using Algebric identity/formula $\cos A\cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$

$$\therefore \boxed{e_{o/p} = \frac{mA_c}{2}\underbrace{\left[\cos(\omega_c+\omega_m)t\right]}_{\text{USB-Compont}} + \frac{mA_c}{2}\underbrace{\left[\cos(\omega_c-\omega_m)t\right]}_{\text{LSB-Compont}}}.$$

By observation of corresponding spectrum of Fig 4 or $e_{o/p} = \frac{mA_c}{2}\left[\cos 2\pi(f_c+f_m)t\right] + \frac{mA_c}{2}\left[\cos 2\pi(f_c-f_m)t\right]$



No Carrier

Fig 4: Frequency Spectrum of a DSB-SC signal.
with $B.W = (f_c+f_m)-(f_c-f_m) = 2f_m = (BW)_{\text{DSB-Full Carrier}}$

$e_{o/p}$ is Hence a DSB-Full carrier which can be transmitted over Comm Channel after having successfully suppressed carrier and hence saving 66.6% of P_T power

So we have achieved power saving but no B.W reduction. Q.

Method 2: Using a Carrier Suppression filter Method

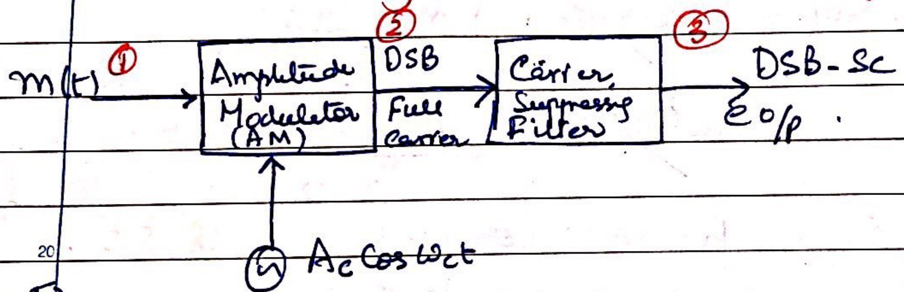


Fig (4): Filter Method.

This is by far the simplest method of generating DSB-SC from Amplitude Modulator output.

Here Filter is used to suppress the carrier at fc.

Position ①:- Message Signal $m(t) = m \cos(\omega_m)t$.

Position ②:- AM $o/p = A_c[1+m(t)]\cos\omega_c t$.

LO generated $A_c \cos \omega_c t$.

$\therefore$ AM $o/p = \cancel{A_c \cos\omega_c t} + \frac{mA_c}{2}\cos(\omega_c+\omega_m)t + \frac{mA_c}{2}\cos(\omega_c-\omega_m)t$

Position ③ :- Filter suppresses $A_c \cos \omega_c t$

After suppression. DSB-SC $= \frac{mA_c}{2}\cos(\omega_c+\omega_m)t + \frac{mA_c}{2}\cos(\omega_c-\omega_m)t$.

Q. What is the main drawback of DSB-SC generation by Filter method?

The Filter method is Very simple and does not require two AM in balanced configuration as was used in BM configuration. However main drawback is that it is very difficult to construct and produce highly frequency selective Filters at high frequency of f_c. Here we have to block just one f_c component without accidently blocking either (f_c+f_m) USB component nor (f_c-f_m) LSB component.

Q. What is the main drawback of BM method of generation of DSB-SC Signal?

(i) BM requires two AM in perfect balanced Configuration. However no two electronic circuitry can be identical. Therefor $(AM)_1$ and $(AM)_2$ cannot be identical. This leaves some residue of uncancelled carrier in o/p.

(ii) The inverter at i/p is used to invert $m(t)$. This is simple if $m(t)$ is single-tone. In realistic terms $m(t)$ is non periodic meaning it has many frequency components. This implies all components of $m(t)$ have to be inverted by equal measure over the entire spectral range of $m(t)$. This is very difficult to achieve and in most probable case leaves uncancelled carrier residue in o/p.

Detection of DSB-SC at Rx.

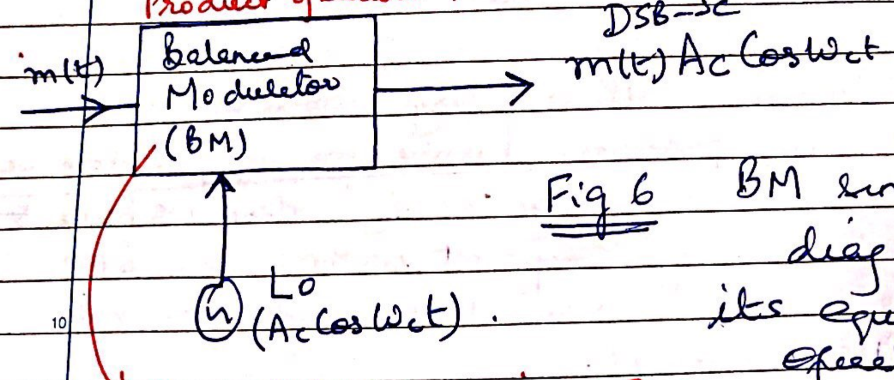
The Balanced Modulator effectively produces an o/p (output) that is the product of two inputs. Refer to Fig(3). where i/p of BM = . $m(t)$

LO input to BM = $A_c \cos 2\pi f_c t$

Output of BM = $A_c\, m(t) \cos 2\pi f_c t$.

This entire operation may be redrawn in simple way as shown in figure 6.

Product operation.



Same BM block diagram of Fig 3.

Fig 6 BM simplified block diag. showing its equivalent product operation.

Thumb rule: 3.8:- Balanced Modulator (BM) ≡ Product Operation

A balanced Modulator performs Multiplication of the two inputs. It is also called a multiplier.

The BM at Tx performs the mathematical product operation of two inputs. At Tx therefore it performs Frequency Translation ie Forward Frequency Translation.

The BM at Rx also similarly performs the mathematical operation of product of two input signals. At the Rx BM is performing Reverse Frequency translation.

From above figure 6 the DSB-SC signal $m(t) A_c \cos(W_c t)$ is Transmitted over Comm Channel. At Rx DSB-SC signal is again supplied to another BM at Rx supplied with identical Carrier to carry out Reverse Frequency Translation at Rx. The Carrier at BM at Tx should be identical in frequency & phase as the Carrier at the point of Reverse Frequency Translation at Rx.

Very important statement.

Q: Explain why carrier used at Rx for DSB-SC detection should be synchronized in both phase and frequency as compared to the carrier used at Tx?

OR

Q. Why should the carrier at the point of forward frequency translation at Tx be synchronized with the carrier at the point of reverse frequency translation at Rx for efficient and effective DSB-SC signal detection?

Assume $m(t) = m\cos\omega_m t$.

Local Oscillator generated Carrier $= A_c\cos\omega_c t$.

where $\omega_c \gg \omega_m$ & $\omega_m = 2\pi f_m t$

$\omega_c = 2\pi f_c t$

Explain this for following cases

Case A At Tx $f_{LO} = A_c\cos\omega_c t$.

At Rx $f_{LO} = \cos\omega_c t$ } ideal Case.

Both are synchronized in frequency & phase.

Case B At Tx $f_{LO} = A_c\cos\omega_c t$.

At Rx $f_{LO} = \cos(\omega_c + \Delta\omega)t \rightarrow$ Frequency Error

freq difference $= \Delta\omega$, Not synchronized in frequency

Case C At Tx $f_{LO} = A_c\cos\omega_c t$.

At Rx $f_{LO} = \cos(\omega_c t + \varphi) \rightarrow$ Phase error

phase difference $= \varphi$; Not synchronized in phase.

HINT OF Solution. Here BM is used both at Tx and at Rx as discussed earlier. The BM at Tx receives two inputs i.e. $m(t)$ & $A_c\cos\omega_c t$ producing DSB-SC signal due to product operation (Frequency translation) of a BM. At Receiver (Rx), the BM receives two inputs again. These will be $(m(t)\, A_c\cos\omega_c t)$

And f_{LO} : In Case A $f_{LO} = A_c \cos\omega_c t$; In Case B $f_{LO} = A_c \cos(\omega_c + \Delta\omega)t$ and Case C $f_{LO} = A_c \cos(\omega_c t + \phi)$.

CASE A

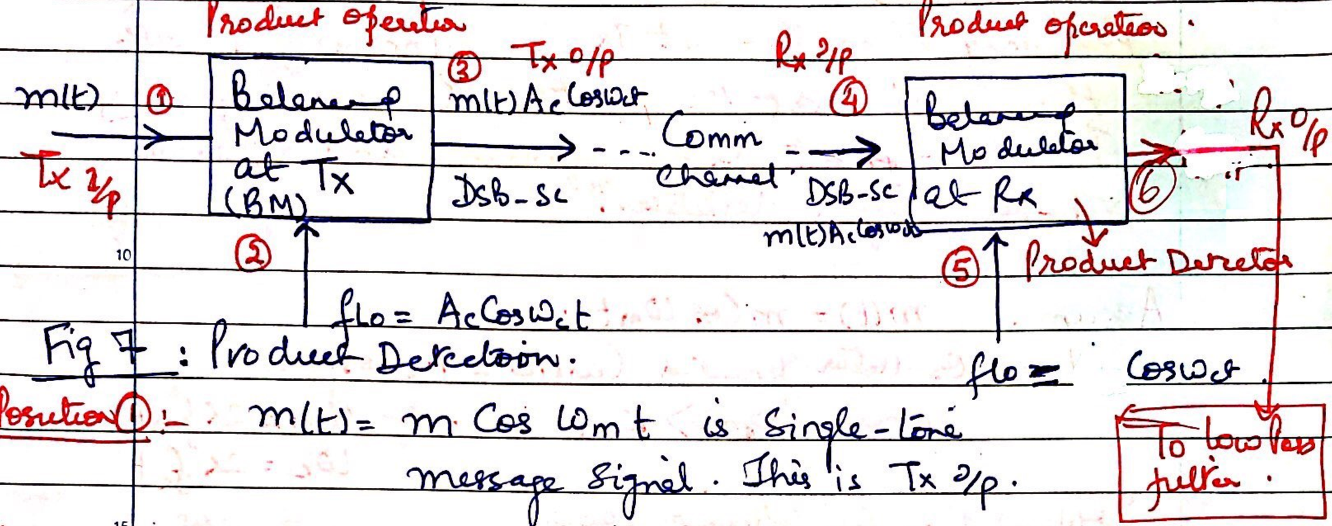


Fig 7 : Product Detection.

Position ① :- $m(t) = m \cos\omega_m t$ is single-tone message signal. This is Tx i/p.

Position ② $f_{LO} \rightarrow$ This is the local oscillator carrier at Tx ie $f_{LO} = A_c \cos(\omega_c t)$ where $\omega_c >> \omega_m$.

Position ③ Output of Tx ; $Tx\ o/p = m(t) \times f_{LO}$.

$$Tx\ o/p = m(t)\, A_c \cos(\omega_c t)$$

$$= [m \cos(\omega_m t)]\, [A_c \cos(\omega_c t)].$$

Use $\cos A \cos B = \frac{1}{2}\cos[A+B] + \frac{1}{2}\cos[A-B]$.

or $$\boxed{Tx\ o/p = \left[\frac{mA_c}{2}\cos(\omega_c + \omega_m)t\right] + \left[\frac{mA_c}{2}\cos(\omega_c - \omega_m)t\right]}.$$

The Tx o/p is thus a DSB-SC signal that is transmitted over Comm. channel.

Position ④ This is the Input of DSB-SC received by Rx.

Hence $Rx\ i/p = m(t)\, A_c \cos\omega_c t$.

$$= \left[\frac{mA_c}{2}\cos(\omega_c + \omega_m)t\right] + \left[\frac{mA_c}{2}\cos(\omega_c - \omega_m)t\right]$$

at the point of reverse frequency translation at Rx.

Position (5) This is the fLO input to the Balanced Modulator at Rx Since this BM at Rx is performing product operation for detection (receiving back m(t)) thus at Rx BM is also called more suitably as Product Detector.

Position (6) This is the Receiver o/p which is product of Rx i/p at position (4) and fLO input at position (5).

$$\therefore \; R_x\, o/p = \left[m(t)(A_c \cos\omega_c t)\right]\left[\cos\omega_c t\right]$$

$$R_x\, o/p = m(t) A_c (\cos^2\omega_c t)$$

Since $\cos(2\theta) = 2\cos^2\theta - 1$

& $\cos^2\theta = \dfrac{\cos(2\theta)+1}{2}$

$$\therefore \; \cos^2(\omega_c t) = \frac{\cos(2\omega_c t)+1}{2}$$

Substitute

Subst. this in Rx o/p.

$$\therefore \; R_x\, o/p = \left[m(t) A_c\right]\left[\frac{\cos(2\omega_c t)+1}{2}\right]$$

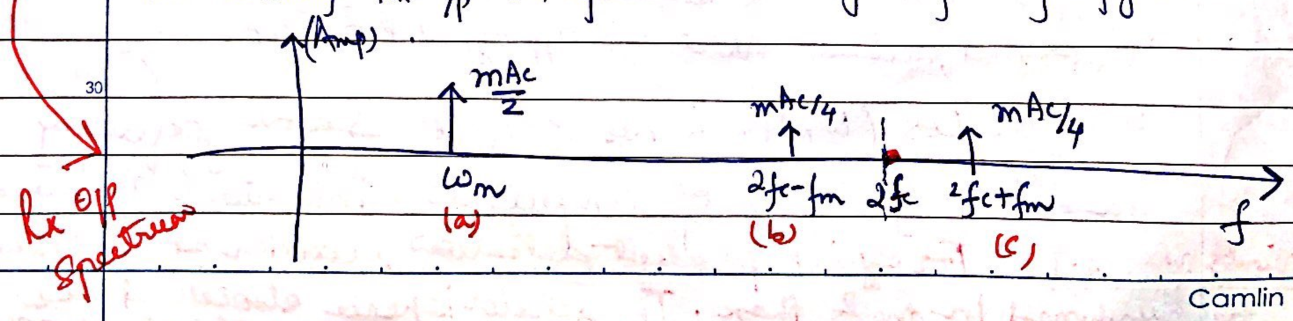
$$\text{or } R_x\, o/p = \left(\frac{A_c}{2}\right)\left[m \cos(\omega_m t)\right]\left[\cos(2\omega_c t)+1\right]$$

$$R_x\, o/p = \frac{mA_c}{2}\left[\cos(\omega_m t)\cos(2\omega_c t)\right] + \frac{mA_c}{2}\cos(\omega_m t)$$

Again using $\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$.

$$\boxed{\therefore \; R_x\, o/p = \frac{mA_c}{4}\Big[\cos(\underbrace{2\omega_c+\omega_m}_{(c)})t\Big] + \frac{mA_c}{4}\Big[\cos(\underbrace{2\omega_c-\omega_m}_{(b)})t\Big] + \underbrace{\frac{mA_c}{2}}_{(a)}\cos(\omega_m t).}$$

On drawing Rx o/p on spectrum we get following figure



The Rx o/p spectrum shows three components.

Component (a) is $\frac{mA_c}{2}$ at fm. This is the recovered and detected message or information signal at Rx. This is the wanted signal.

Component (b) is $\frac{mA_c}{4}$ component at $2f_c - f_m$. This is frequency translated around $2f_c$ below $2f_c$ by fm. This is unwanted high freq. component and can be removed by a low pass filter.

Component (c) is $\frac{mA_c}{4}$ component at $2f_c + f_m$. Again this is described as an unwanted component frequency translated around $2f_c$ above $2f_c$ by fm.

After Low Pass filter :- Rx o/p → $\boxed{\text{LPF } (\omega_m)}$ → m(t)

A low pass filter that block all frequencies above ω_m will cancel component (b) and component (c) at $(2f_c \mp f_m)$ and Residue or it will pass at o/p

$$R_x \text{ LPF o/p} \rightarrow \frac{mA_c}{4} \cos(2\pi f_m)t$$

or

$$\boxed{R_x \text{ LPF o/p} = \frac{mA_c}{4} \cos(\omega_m)t}$$

This is the recovered and detected message signal. Only difference is that at I/p BM (Input Tx Balanced Modulator) message was $m(\cos\omega_m t)$ and now at o/p of LPF message is $\frac{mA_c}{4}\cos(\omega_m)t$. This suggest same frequency and only difference of amplitude. Information is recovered.

Thumb rule 3.9 :- For efficient product detection, carrier at Tx should be synchronized in freq & phase. The detector is hence also called the COHERENT DETECTOR at the point of Reverse Frequency Translation at Rx.

(30) This is also called Frequency Shift of $\Delta\omega$. or frequency error.

CASE B Refer to Same block diag. of Fig (7) of the product detector. Now $f_{LO} = \cos(\omega_c + \Delta\omega)t$.

HINT: Here Tx f_{LO} and Rx f_{LO} differ in Carrier is freq of $\Delta\omega$. Repeat as in Case (A) to describe Position (1), Position (2) Position (3) & Position (4). Now with $f_{LO} = \cos(\omega_c + \Delta\omega)t$ Redefine Position (5) signal and finally Position (6) signal before LPF and After LPF.

HINT: Show that
$$Rx(o/p) = \frac{mA_c}{4}\left[\cos(2\omega_c + \omega_m + \Delta\omega)t\right] + \frac{mA_c}{4}\left[\cos(2\omega_c - \omega_m + \Delta\omega)t\right] + \frac{mA_c}{4}\left[\cos(\omega_m - \Delta\omega)t\right] + \frac{mA_c}{4}(\omega_m + \Delta\omega)t$$

Q. Then What will be LPF o/p? What does that o/p imply.

Q. Is it an effective detection procedure?

CASE C This is also called Phase shift ϕ. or phase error. Refer to same block diag of Fig (7) of the product detector. Now $f_{LO} = \cos(\omega_c t + \phi)$ ie phase error in carrier. Repeat to describe signal at Position (1), Position (2), Position (3) & Position (4). Now with $f_{LO} = \cos(\omega_c t + \phi)$. redefine Position (5) signal and finally Position (6) signal before LPF and after LPF.

HINT: Show that
$$Rx(o/p) = \frac{mA_c}{4}\left[(2\omega_c + \omega_m)t + \phi\right] + \frac{mA_c}{4}\left[(2\omega_c - \omega_m)t + \phi\right] + \frac{mA_c}{4}\cos(\omega_m t + \phi) + \frac{mA_c}{4}\cos(\omega_m t - \phi).$$

Q. Then What will be LPF o/p? What does that O/p imply?

Q. Is it an Effective detection procedure.

Scanned with CamScanner