2.5.3 Fourier Transform Formula

If $f(t)$ and $F(j\omega)$ form a Fourier transform pair then we may write a more modified form of

$$F(f) \rightleftharpoons f(t)$$ as mentioned on pg 23

Generally it will be more appropriate to express the frequency domain function of (ω) the angular frequency so that

$$F(j\omega) = \mathcal{F}[f(t)] \quad \ldots\ldots \text{Eq. } (2.9.a)$$

Frequency domain function — Fourier Transform Operation — Time domain function

or

$$f(t) = \mathcal{F}^{-1}[F(j\omega)] \quad \ldots (2.9.b)$$

Where $$F(j\omega) = \int_{-\infty}^{\infty} f(t) \exp[-j\omega t]\, dt \quad \ldots (2.9.c)$$

and $$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) \exp[j\omega t]\, d\omega \quad \ldots (2.9.d)$$

So that $\boxed{F(j\omega) \rightleftharpoons f(t)}$ forms a Fourier transform pair.

Thus $F(j\omega)$ is the Fourier Integral of $f(t)$
$F(j\omega)$ represents a continuous amplitude spectrum
$\varphi(j\omega)$ represents a continuous phase spectrum

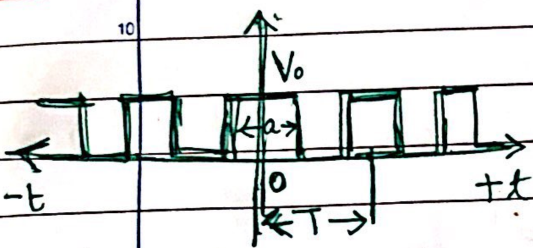
where

$F(j\omega) = F(-j\omega)$ . . . . Even Symmetry.
$\varphi(-j\omega) = -\varphi(j\omega)$ . . . . Odd Symmetry.

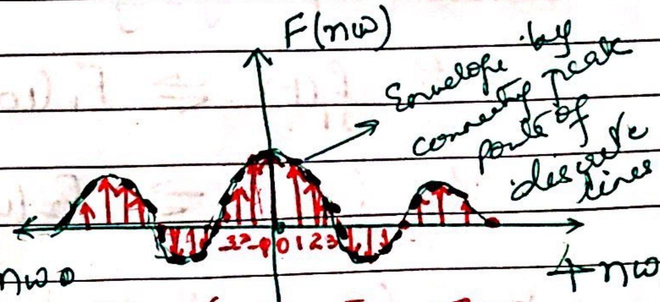
& $F(j\omega) = |F(j\omega)| \exp[j\varphi(\omega)]$ ... (2.9.e)

The shape of continuous amplitude spectrum and phase spectrum for a non-periodic function $f(t)$ is identical with the envelopes of the discrete amplitude & phase line spectra for the same pulse recurring.

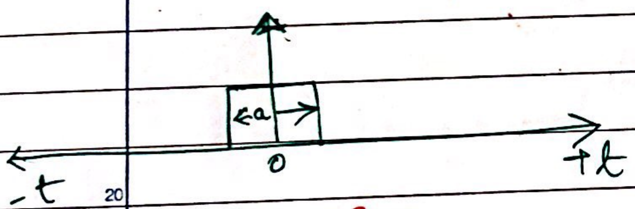
Thumbs rule # 2.12



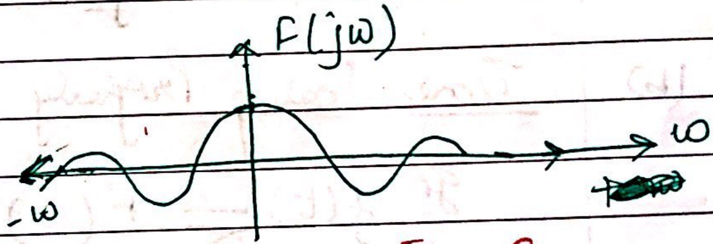
Periodic square waveform
Duty factor = $\frac{a}{T}$.



Discrete spectrum where
$\Delta\omega = \frac{2\pi}{T} = \frac{2\pi}{Na}$. if $\frac{a}{T} = \frac{1}{N}$



Non-periodic square pulse.
$\frac{a}{T} = \frac{1}{N}$ when $T \to \infty$



Continuous spectrum same shape as of envelope of Discrete spectra

2.5.4 Properties of Fourier Transform [Reference "Simon Haykens"]

Following are some of the properties of Fourier Transform
(a) Linearity Property
(b) Scaling Property
(c) Time Reversal
(d) Time shifting
(e) Frequency shifting

(f) Duality
(g) Differentiation
(h) Area under ~~f(t)~~ $f(t)$
(i) Area under $F(\omega)$
(j) ~~(j)~~

Statements & Definitions of these properties.

(a) Linearity Property

If $f_1(t) \rightleftharpoons F_1(\omega)$.

& $f_2(t) \rightleftharpoons F_2(\omega)$

Then for constants 'a' & 'b'

$$a f_1(t) + b f_2(t) \rightleftharpoons a F_1(\omega) + b F_2(\omega)$$

(b) Time Scaling Property

If $f(t) \rightleftharpoons F(\omega)$

$$f(at) \rightleftharpoons \frac{1}{|a|} F(\omega/a)$$

If $f(at)$ represents a fn $f(t)$ compressed in time by a factor 'a' $(a>1)$ then $F(\omega/a)$ represents $F(\omega)$ expanded in frequency by same factor ~~(....)~~

(c) Time Reversal Property

If $f(t) \rightleftharpoons F(\omega)$

then $\mathcal{F}[f(-t)] \rightleftharpoons F(-\omega)$.

(d) Time Shifting Property

If $f(t) \rightleftharpoons F(\omega)$

then $f(t-t_0) \rightleftharpoons F(\omega)\exp[-j\omega t_0]$.

(e) Frequency Shifting Property

If $f(t) \rightleftharpoons F(\omega)$

$[f(t)\exp[j\omega_0 t]] \rightleftharpoons F(\omega-\omega_0)$

(f) Duality Property:

If $f(t) \rightleftharpoons F(\omega)$

then $[F(t)] \rightleftharpoons 2\pi f(-\omega)$

(g) Differentiation Property

If $f(t) \rightleftharpoons F(\omega)$

then $\left[\frac{d}{dt} f(t)\right] \rightleftharpoons j\omega F(\omega)$.

(h) Frequency Differentiation:

If $f(t) \rightleftharpoons F(\omega)$

then $[-jt]f(t) \rightleftharpoons \frac{d}{d\omega} F(\omega)$.

(i) **Area under f(t).**

If $f(t) \rightleftharpoons F(\omega)$

then $\int_{-\infty}^{\infty} f(t)\,dt \rightleftharpoons F(0)$

Thus the area under a function g(t) is equal to value of the Fourier Transform $F(\omega)$ at $\omega = 0$. This value is obtained by simply putting $\omega = 0$ in $F(\omega)$.

(j) **Area under F(ω)**

If $f(t) \rightleftharpoons F(\omega)$

then $f(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\,d\omega$.

The value of the function at $t = 0$ is $f(0)$. This is equal to the area under its Fourier Transform. This is the value that is obtained by putting $t = 0$ in the formula that defines the Fourier inverse Transform.

Q.17. Same Question as already stated on Pg 50 of lecture dt 29.04.20.

Q.18 Give at least two examples that show application of each property of Fourier Transform. Two examples each from property (a) to property (i) means that total number of examples with illustration of function in time domain & frequency domain required is Eighteen.
[(a) – 2 examples . . . . (i) . . 2 examples]
Show illustrations with examples wherever necessary.

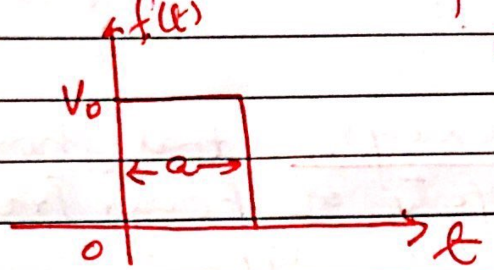
Q.19. With reference to "Network Analysis" by Van Valkenburg Find the fourier transform of following functions. (Also draw the functions in time domain & frequency domain).

(1) A decaying exponential
(2) A rising exponential
(3) A double exponential
(4) A delta function
(5) $f(t) = 1$
(6) $f(t) = \exp(j\omega_0 t)$
(7) $f(t) = \exp(-j\omega_0 t)$

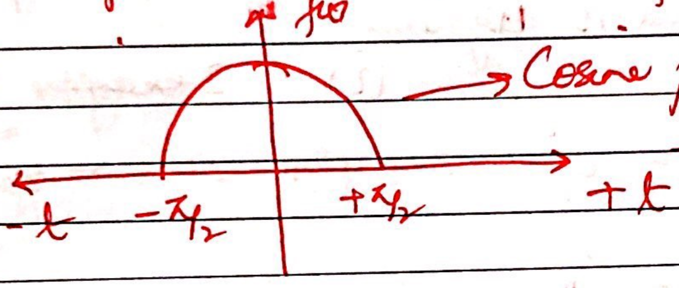
Q.20 Obtain the Fourier transform with neat illustrations of frequency domain & Time domain of :—
(1) $f(t) = \cos(\omega_0 t)$
(2) $f(t) = \sin(\omega_0 t)$

Q.21. Determine the Fourier Transform of an impulse Train

Q.22. Derive Fourier Transform & draw relevant time domain & frequency domain function of following non-periodic function

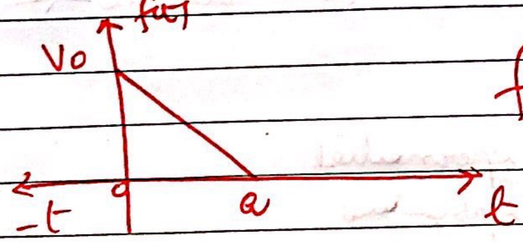


Q.23. Repeat above question for following pulse.



→ Cosine pulse from $t = -\tau/2$ to $t = +\tau/2$

Q.24 Repeat Q.22 for –



$f(t) = -\frac{V_0}{a}(t-a)$ ..... $0 < t < a$.

Note 1: – Last date of submission from Q.17 to Q.24 → 05.05.20 [Tuesday].

Note 2:– This is <u>End</u> of Topic(2)

Any doubts to be clarified can be conveyed directly or through CR's.

<u>TOPIC#2 ENDS.</u>