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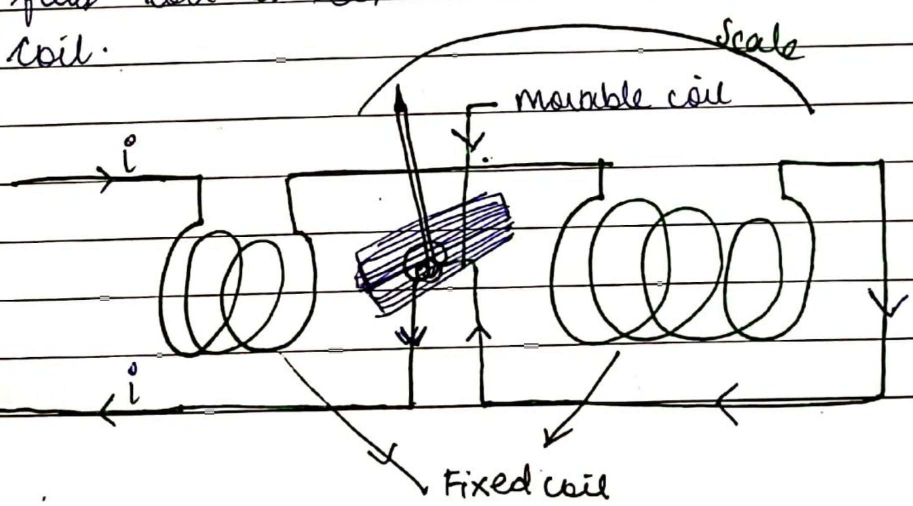
ELECTRODYNAMOMETER TYPE OF INSTRUMENT

Electrodynamometer type of instruments are used as ac Voltmeters & ammeters in range of power frequencies and lower part of audio freq range.
They are used as wattmeter; Varmeter & with some modifications as power factor meters & freq. meter.

Operating Principle

Consider how PMMC instrument would behave on a.c. It would have torque in one direction during one half of the cycle and an equal effect during other half of the cycle.
For an ordinary meter, inertia on power frequencies is so great, that the pointer doesn't go very far in either direction but merely stays around zero.

If however, we were to reverse the direction of the field flux each time, the current through the movable coil reverses, the torque would be produced in the same direction for both halves of the cycle.
The field can made to reverse simultaneously with the current in the movable coil if the field coil is connected in series with the movable coil.



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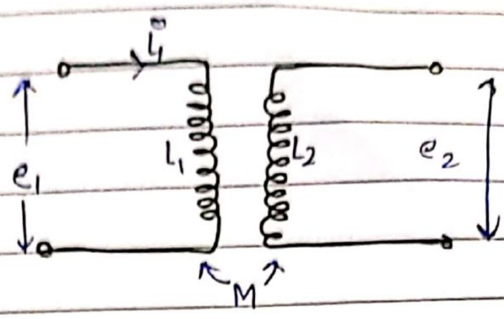
TORQUE EQUATION



i_1 = instant. current in fixed coil; A

i_2 = instant current in moving coil; A

L_1 = self inductance of fixed coils; H
L_2 = " " " moving " ; H
M = mutual inductance between fixed & moving coils ; H.

Flux linkage of coil 1, $\lambda_1 = L_1 i_1 + M i_2$
" " " " 2; $\lambda_2 = L_2 i_2 + M i_1$

Electrical input energy

$$e_1 i_1 dt + e_2 i_2 dt = i_1 d\lambda_1 + i_2 d\lambda_2$$

as $e_1 = \frac{d\lambda_1}{dt}$ & $e_2 = \frac{d\lambda_2}{dt}$

$$i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_2)$$

$$\Rightarrow i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \quad \text{--- (1)}$$

Energy stored in the magnetic field

$$= \frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M$$

Change in energy stored $= d\left(\frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M\right)$

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$\Rightarrow i_1 L_1 di_1 + \left(\frac{i_1}{2}\right)^2 dL_1 + i_2 L_2 di_2 + \left(\frac{i_2}{2}\right)^2 dL_2 + i_1 M di_1$

$+ i_2 i_2 dM \longleftarrow (2)$

From principle of conservation of energy
Total electrical input energy = change in energy + mechanical energy

$\therefore$ Mechanical energy = (1) - (2)

$$ME = \left(i_1 L_1 \cancel{di_1} + i_1^2 dL_1 + i_1 \cancel{i_2} dM + i_1 \cancel{M} di_2 + i_2 \cancel{L_2} di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 \cancel{M} di_1\right)$$

$$- (i_1 L_1 \cancel{di_1}) - \frac{i_1^2}{2} dL_1 - i_2 \cancel{L_2} di_2 - \frac{i_2^2}{2} dL_2$$

$$- i_1 \cancel{M} di_2 - i_2 \cancel{M} di_1 - i_1 \cancel{i_2} dM$$

$$\therefore ME = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$$

Now the self inductances L_1 & L_2 are constant
& $\therefore dL_1, dL_2 = 0$.

$$\therefore ME = i_1 i_2 dM$$

Suppose T_i = instantaneous deflecting torque
$d\theta$ = change in deflection.
Then work done = $T_i \cdot d\theta$

Thus we have, $T_i \, d\theta = i_1 i_2 \, dM$

or $\boxed{T_i = i_1 i_2 \frac{dM}{d\theta}}$ — (iii)

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Operation with DC

let I_1 = current in fixed coils, A

I_2 = " " moving coils, A

$\therefore T_d = I_1 I_2 \cdot \frac{dM}{d\theta}$ ($\because$ from (iii))

This deflecting torque deflects the moving coil to such a position where the controlling torque of the spring = deflecting torque.

Suppose θ = final steady deflection.

$\therefore$ Controlling torque, $T_c = K\theta$

At final steady position, $T_d = T_c$

$I_1 I_2 \frac{dM}{d\theta} = K\theta$ or $\boxed{\theta = \frac{I_1 I_2}{K} \frac{dM}{d\theta}}$

Operation with AC

i_1, i_2 be the instantaneous values of currents carried by the coils.

$T_i = i_1 i_2 \cdot \frac{dM}{d\theta}$

Average deflection torque over a complete cycle $T_d = \frac{1}{T} \int_0^T T_i \cdot dt$

$= \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T i_1 i_2 \, dt$

where T = time period for one complete cycle.

Sinusoidal current

Let phase angle between two sinusoidal currents i_1, i_2 be ϕ

$$i_1 = I_{m_1} \sin \omega t$$
$$i_2 = I_{m_2} \sin(\omega t - \phi)$$

Average deflecting torque

$$T_d = \frac{dM}{d\theta} \frac{1}{T} \int_0^T i_1 i_2 \, dt$$

$$= \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T I_{m_1} \sin \omega t \; I_{m_2} \sin(\omega t - \phi) \cdot dt$$

$$= \frac{dM}{d\theta} \cdot I_{m_1} I_2 \frac{1}{2\pi} \int_0^{2\pi} \sin \omega t \sin(\omega t - \phi) \, d(\omega t)$$

$$= \frac{I_{m_1} I_{m_2}}{2} \cos\phi \cdot \frac{dM}{d\theta}$$

$$\boxed{T_d = I_1 I_2 \cos\phi \frac{dM}{d\theta}}$$ where I_1 & I_2 are rms values.

At steady state, $T_d = T_c = K\theta$

$$I_1 I_2 \cos\phi \frac{dM}{d\theta} = K\theta$$

deflection $\boxed{\theta = \frac{I_1 I_2 \cos\phi}{K} \cdot \frac{dM}{d\theta}}$

Thus for sinusoidal currents, deflection is determined from the products of rms values of currents & cosine of angle b/w them.

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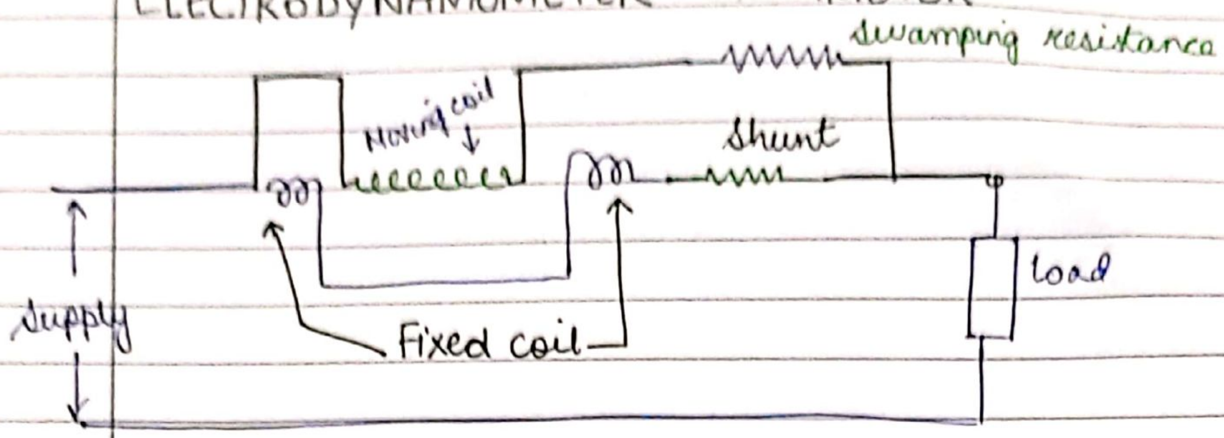
ELECTRODYNAMOMETER AMMETER



The fixed and moving coils are connected in series & ∴ carry the same current

$$I_1 = I_2 = I$$

$$\therefore \; T_d = I^2 \, dM/d\theta$$

and deflection $\boxed{\theta = \frac{I^2}{K} \frac{dM}{d\theta}}$

This arrangement ~~for~~ is suitable for ammeters having a range ~~upto 200mA~~ greater than 200mA.

Moving coil is connected in series with a swamping resistance across a shunt together with fixed coils.

In order, that ammeter may indicate correctly at all frequencies, the currents in fixed & moving coils must be in phase.

This requires time constant L/R of two branches to be equal, otherwise currents in two branches will not be independent of frequency.

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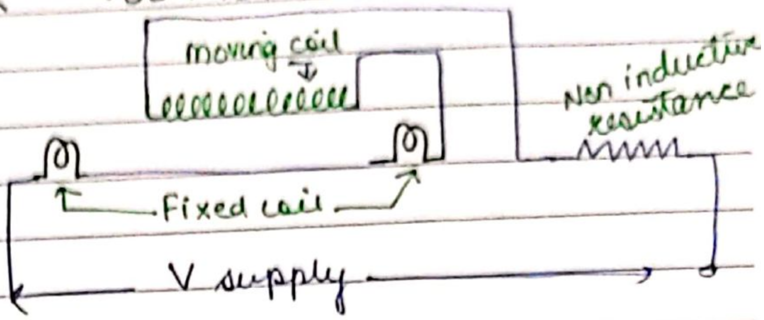
Using this arrangement, we can easily make $\left(\frac{L}{R}\right)$ ratio equal for both branches.

ELECTRODYNAMOMETER VOLTMETER

The electrodynamometer is used as a voltmeter by connecting the fixed and moving coils in series with high non inductive resistance.



$$T_d = I_1 I_2 \cos\phi \; dM/d\theta$$

$$I_1 = I_2 = \frac{V}{Z} \quad \text{and} \quad \phi = 0$$

$$T_d = \left(\frac{V}{Z}\right)\cdot\left(\frac{V}{Z}\right)\frac{dM}{d\theta}$$

Z : Impedance of instrument ckt.

$$T_d = \frac{V^2}{Z^2}\cdot\frac{dM}{d\theta}$$

$$\therefore \text{Deflection } \theta = \frac{V^2}{KZ^2}\cdot\frac{dM}{d\theta}$$

Most accurate type of ac voltmeters.

Sensitivity low as compared to dc instrument (10 − 30 Ω/V).

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Error incorporated

(i) Low torque / weight ratio (causes friction losses)
(ii) Frequency (due to self reactance of coil)
(iii) Eddy currents (This produces torque becoz of coupling)
(iv) External magnetic field (Use metal shields)
(v) Temperature change
(Self heating of coils becoz of high I)

USE ON DC and AC

When used on ac, instantaneous torque is proportional to i^2 (for ammeter).

As I varies → Torque varies → but direction remains same. Pointer takes up a position where average torque = controlling torque.

If initially calibrated on dc, instrument reads rms value.

Shape of Scale

$\theta \propto I^2 \cdot \frac{dM}{d\theta}$ for an ammeter.

$\theta \propto V^2 \cdot \left(\frac{dM}{d\theta}\right)$ " voltmeter.

Thus scale is not uniform. These have square law response.