(3) Superposition Theorem

The Superposition Principle states that the voltage across (or current through) an element in a linear circuit is the algebric sum of the voltages across (or currents through) that element due to each independant source acting alone.

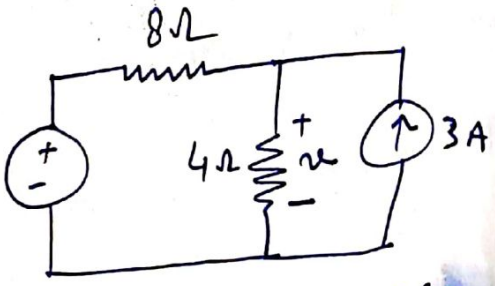
The principle of superposition helps us to analyse a linear circuit with more than one independant source by calculating the contribution of each independant source seperately.

I → We consider one independent source at a time while all other independent sources are suppressed. By suppressing a source we mean voltage source is shortcircuited (Replaced by 0V) and every current source (open circuited) Replaced by 0 I or ∞ resistance)

II. Dependant sources are kept intact because they are controlled by circuit variables.

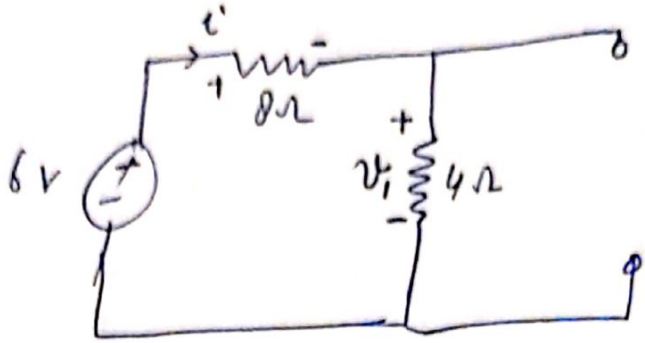
Example:-

use superposition to find v;



Let the Contribution to 'V' due 6V by v_1 and due to 3A by v_2;

Step i Suppress current source (3A)., we are left with.

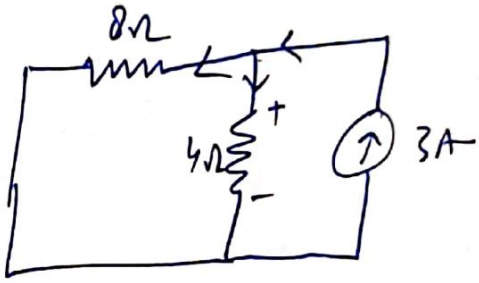


$$i' = \frac{6}{8+4}$$

$$= \frac{6}{12} = 0.5\,A;$$

$$v_1 = 0.5 \times 4 = 2.0V.$$

Step ii: Now suppress 6V and keep (3A) live.



Current through 4Ω

$$= 3A \times \frac{8}{8+4}$$

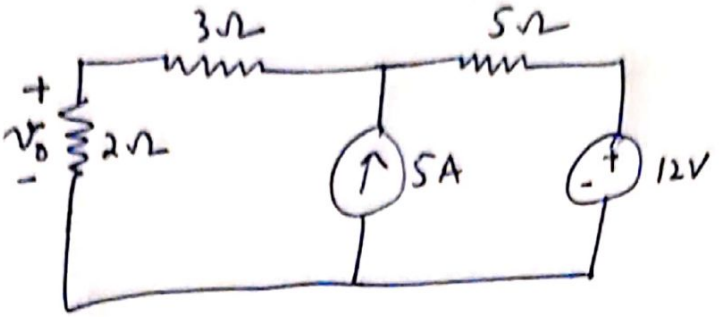
$$= \frac{3 \times 8}{12} = 2A.$$

$$\therefore v_2 = 2 \times 4 = 8V. \quad [\text{with polarity shown}].$$

$$\therefore v = v_1 + v_2 \quad [\text{both in same direction}]$$

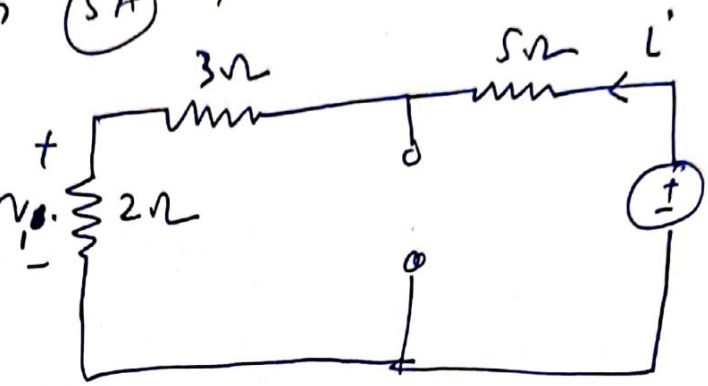
$$\boxed{v = 2.0 + 8V = 10V.}$$

Exp.



Find v_0 using superposition theorem.

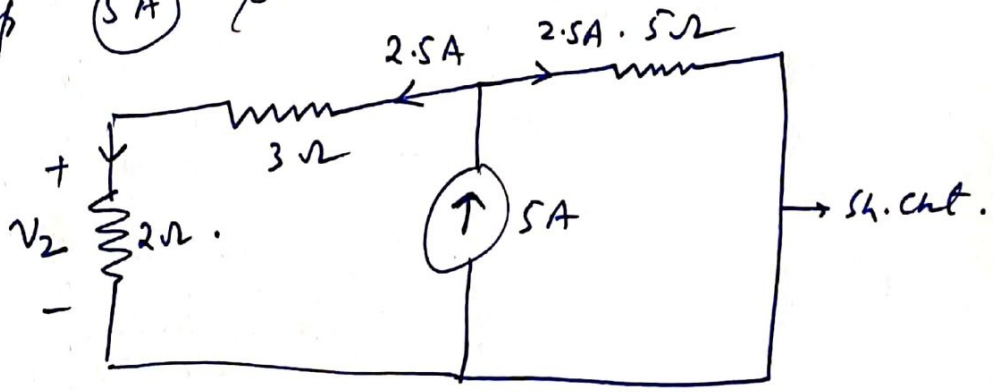
Step I: Suppress (5A) source & keep 12V source live.



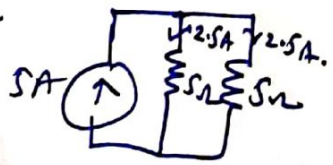
$$i = \frac{12}{2+3+5} = \frac{12}{10} = 1.2A.$$

$\therefore v_1 = 1.2 \times 2 = 2.4\,V$ [Polarity shown].

Step ii Keep (5A) live & suppress 12V source.



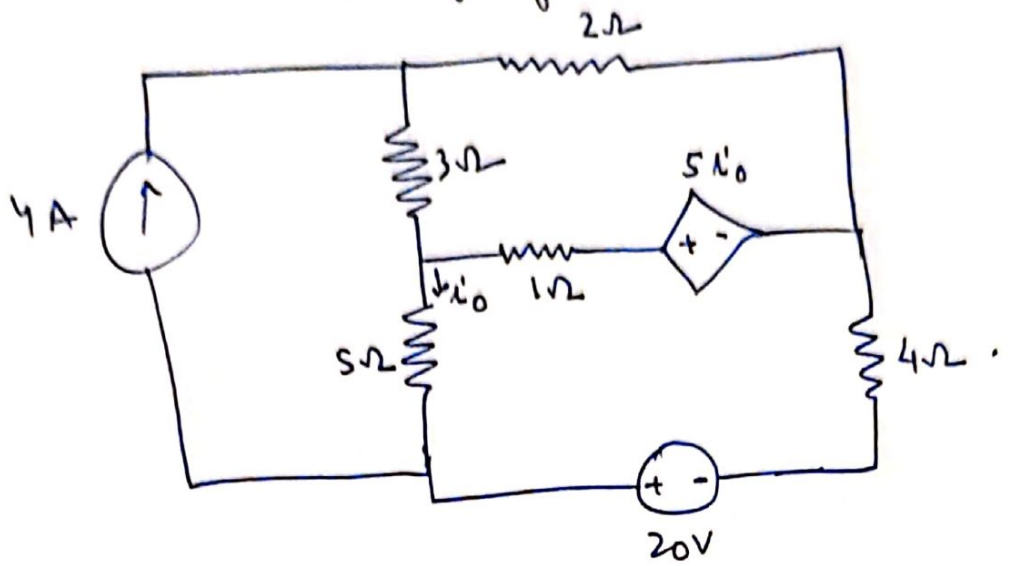
$v_2 = 2.5 \times 2 = 5.0\,V$ [Polarity + − top & bottom].



$\therefore V = 2.4 + 5.0 = 7.4\,V.$

$\therefore \boxed{v_0 = V = 7.4\,V.}$ ✓

Example: Comprising of both dependant & independant sources.



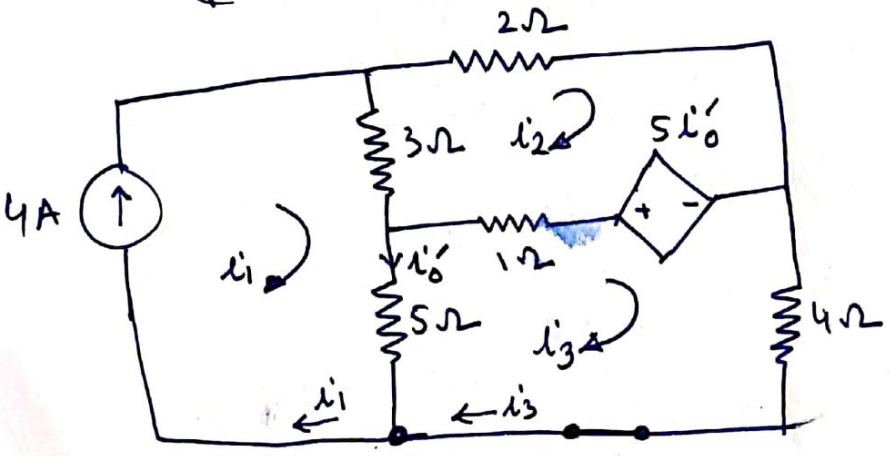
" Find i'_o using Superposition Theorem "

The dependent source $5i'_o$ has to be left intact.

Let $i'_o = i'_o{}' + i'_o{}''$

where $i'_o{}'$ = Contribution due to (4A) current source;

& $i'_o{}''$ = " " (20V) voltage source;



$i'_1 = 4A$; ① → loop 1:

loop 2: $(2+3+1)i'_2 - 3i'_1 - i'_3 = 5i'_o$

$-3i'_1 + 6i'_2 - i'_3 - 5i'_o = 0$ ②.

Loop 3:

$$-5i_1 - i_2 + 10i_3 = -5i_0' \quad \text{---(3)}.$$

KCL at node 0; $i_0' + i_3 = i_1$

$$\therefore i_3 = i_1 - i_0' \quad \text{---(4)}.$$

$$\boxed{i_3 = 4 - i_0'}$$

Substituting eqn (1) & (4) in (2) & (3), we get:

eqn (2):

$$-3i_1 + 6i_2 - (4 - i_0') - 5i_0' = 0$$

$$-5i_1 - i_2 + 10(4 - i_0') + 5i_0' = 0$$

$$-12 + 6i_2 - 4 + i_0' - 5i_0' = 0$$

$$6i_2 - 16 - 4i_0' = 0$$

$$\boxed{3i_2 - 2i_0' = 8} \quad \text{---(5)};$$

eqn (3):

$$-5 \times 4 - i_2 + 10(4 - i_0') + 5i_0' = 0.$$

$$-20 - i_2 + 40 - 10i_0' + 5i_0' = 0$$

$$-i_2 + 20 - 5i_0' = 0.$$

$$\boxed{i_2 + 5i_0' = 20} \quad \text{---(6)}.$$

Solve (5) & (6); to get $i_0' = ?$

from (6) $i_2 = 20 - 5i_0'$

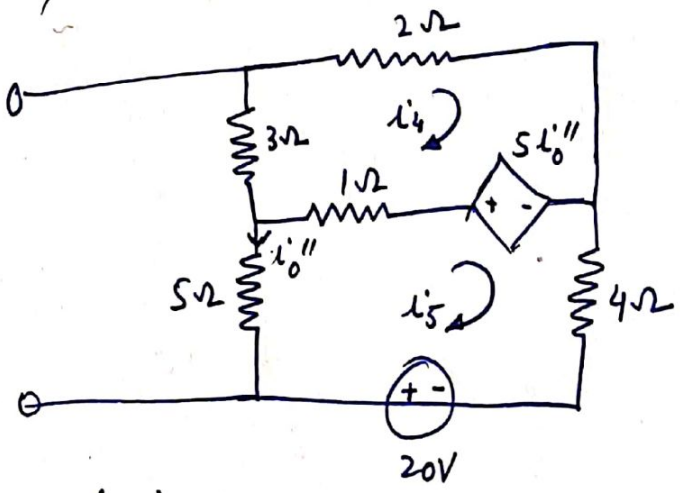
from (5); $3(20 - 5i_0') - 2i_0' = 8$

$60 - 15i_0' - 2i_0' = 8$

$-17i_0' = 8 - 60 = -52$

$$i_0' = \frac{52}{17} \text{ A} \quad \text{---(7);}$$

Now Let us suppress 8 (4A) source and keep 20V source like, we get;



Apply mesh analysis:-

Mesh 4; $6i_4 - i_5 = 5i_0''$ ——— (8)

Mesh 5; $-i_4 + 10i_5 = 20 - 5i_0''$ ——— (9).

Also; $i_5 = -i_0''$ ——— (10);

From eqn (8); $6i_4 - (-i_0'') = 5i_0''$

$6i_4 + i_0'' - 5i_0'' = 0$

$$6i_4 - 4i_0'' = 0 \quad \text{——— (11)};$$

From (3);

$$-i_4 + 10(i_0'') = 20 - 5i_0''$$

$$-i_4 = 20 - 5i_0'' + 10i_0''$$

$$-i_4 = 20 + 5i_0'' \quad \text{——— (12)};$$

From (11); $6(-20 - 5i_0'') - 4i_0'' = 0$

$$-120 - 30i_0'' - 4i_0'' = 0$$

$$-120 - 34i_0'' = 0$$

$$\boxed{i_0'' = \frac{-120}{34} = -\frac{60}{17}\ A.} \quad \text{——— (13)}.$$

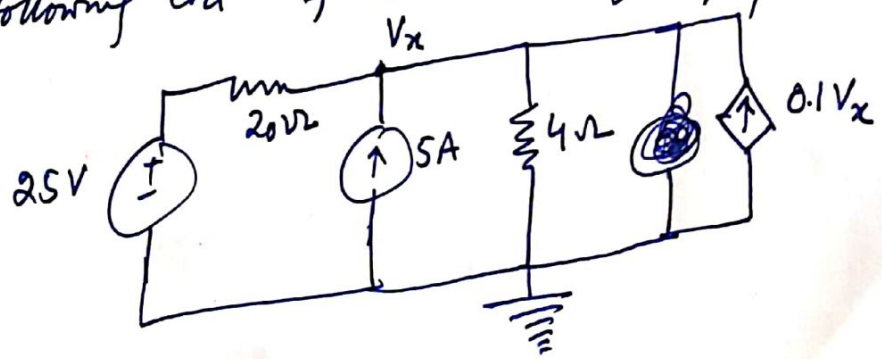
$\therefore i_0 = i_0' + i_0''$

$$= \frac{52}{17} - \frac{60}{17}$$

$$\boxed{i_0 = \frac{52 - 60}{17} = \frac{-8}{17} = -0.4706\ A.}$$

——— o ———

Expl. For the following ckt find V_x using superposition principle.



$$\boxed{V_x = 31.25\ V}$$