Topic Date

BALLISTIC GALVANOMETER

It is used for measurement of charge passed through it.

In magnetic measurements, this charge is due to instantaneous emf induced in a search coil connected across the Ballistic galvanometer.

The instantaneous emf is induced when the flux linking with the search coil is changed.

Quantity of electricity passing through galvanometer is $\propto$ emf induced and hence to the change in flux linking with the search coil.

Construction : The ballistic galvanometer is of d' Arsonval type. But it doesn't show a steady deflection due to transitory nature of the current passing through, but it oscillates with decreasing amplitude of the first deflection or swing being proportional to the charge passing.

This proportionality b/w first deflection & charge passing holds good only if the whole charge passes through the galvanometer coil before any appreciable deflection of the coil takes place.

This condition can be satisfied if the time taken by the charge to pass is small & the time period of the undamped oscillations of the galvanometer is large.

Topic   Date

Time period for undamped oscillations of a galvanometer is

$$T_0 = 2\pi\sqrt{J/K}$$

J : inertia constant   K : control constant.

If J ↑ , K ↓ ∴ T ↑ (usually 10-15 sec

The above conditions can be satisfied by attaching small weights to the moving system in order to increase the J & by using suspension of smaller stiffness so as to decrease K.

Damping of galvanometer should be small in order that the amplitude of first swing is large. After first swing, electromagnetic damping may be used to bring the coil rapidly to rest. This can be done by having a key connected across the galvanometer terminals ; this key shorts the galvanometer coil when closed.

Theory : The moving system deflects only after the charge has completely passed through the meter. The moving system has to deflect in order that the energy imparted to it by the charge is dissipated gradually in friction and electromagnetic damping.

But during the actual motion, there is no deflecting torque as there is no current through the coil.

(3)

Topic Date

Q : charge to be measured.

$Q = \int i\,dt$ or $i = \frac{dQ}{dt}$.

let the instant of time during which the charge Q passes be defined as the time b/w $t = t_0 = 0$ and $t = t_1$.

B/w t_0 to t_1, there is no motion of the coil of the galvanometer & equation of motion is

$$J \frac{d\theta^2}{dt^2} + D\frac{d\theta}{dt} + K\theta = Gi$$

$$J \frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = \frac{G \cdot dQ}{dt}$$

For $\frac{d^2\theta}{dt^2} = \left(\frac{D}{J}\right) \cdot \frac{d\theta}{dt} + \left(\frac{K}{J}\right) \cdot \theta = \frac{G}{J} \cdot \frac{dQ}{dt}$

Integrating above.

(1) — $$\left.\frac{d\theta}{dt}\right|_{t_0=0}^{t_1} + \left.\frac{D}{J}\theta\right|_{t=0}^{t_1} + \frac{K}{J}\int_0^{t_1} \theta \cdot dt = \frac{G}{J} \cdot Q$$

But deflection $\theta = 0$ ~~also~~ during $(t_0$ to $t_1)$.

$\therefore$ (1) becomes $\frac{d\theta}{dt} = \frac{G}{J} Q$.

After t_1, no current passes through the coil of the galvanometer & hence deflecting torque is zero.

Citizen

Topic Date

∴ Eq. of motion after t_1 =

$$J \cdot \frac{d^2\theta}{dt^2} + D \cdot \frac{d\theta}{dt} + K\theta = 0$$

The sol. = $\theta = F e^{-\frac{D t}{2J}} [\sin(\omega_d t + \phi)] + \theta_F$

$\theta_F = 0$ as galvanometer shows no steady state deflection.

$$\therefore \theta = F e^{-\frac{D t}{2J}} [\sin(\omega d t + \phi)] \quad (2)$$

Now we try to eleminiate F & ϕ by applying boundary conditions.

Diff (2) wrt t;

$$\frac{d\theta}{dt} = F \omega_n e^{-D/2J\, t} [\cos(\omega_n t + \phi)]$$
$$- F e^{-D/2J\, t} \frac{D}{2J} [\sin(\omega_n t + \phi)] \quad (3)$$

Initial condition at $t = 0$ are $\theta = 0$; $\frac{d\theta}{dt} = \frac{G Q}{J}$

∴ From (2) we get

$0 = F \sin\phi$ ∴ $\phi = 0$

From (3) we get $F = \frac{G}{J} Q$.

Topic ______ Date ______

$\therefore$ Eq (I) becomes

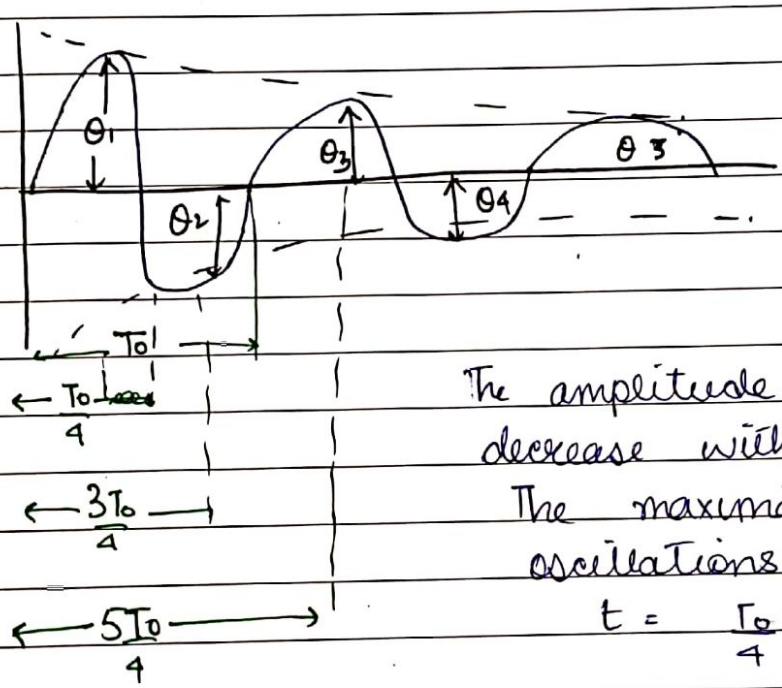
$$\theta = \frac{G}{J\omega_n} Q \cdot e^{-\frac{D}{2J}t} \sin \omega_n t$$

$$= \frac{G}{J}\sqrt{\frac{J}{K}} \; Q \cdot e^{-\frac{D}{2J}t} \sin \sqrt{\frac{K}{J}}\, t$$

$$= A Q \, e^{-\frac{D}{2J}t} \sin \left(\frac{2\pi}{T_0}\right) \cdot t$$

Where $A = \frac{G}{J}\sqrt{\frac{J}{K}}$.

Thus charge is proportional to deflection at any instant.
The motion of galvanometer is oscillatory with a decreasing amplitude.



deflection time curve for BG.

The amplitude of oscillations decrease with time.
The maxima of successive oscillations occurs at
$t = \frac{T_0}{4}, \frac{3T_0}{4}, \frac{5T_0}{4}$.

Topic Date

$$\theta_1 = KQ e^{-D/2J\left(\frac{T_0}{4}\right)} \sin\left(\frac{2\pi}{T_0}\right)\left(\frac{T_0}{4}\right)$$

$$= KQ e^{-\frac{D}{2J}\cdot\frac{T_0}{4}}$$

Putting the value of $T_0 = 2\pi\sqrt{J/K}$

$$\theta_1 = AQ e^{\left(\frac{-D}{2J}\cdot\frac{2\pi}{4}\sqrt{\frac{J}{K}}\right)}$$

$$\theta_1 = AQ e^{\left(-\pi/4 \frac{D}{\sqrt{JK}}\right)} \quad \text{——— (1)}$$

Introducing the concept of logarithmic decrement. We know that

$$\lambda = \frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\frac{T_0}{T_d} = \sqrt{1-\zeta^2} \qquad \therefore \lambda = \zeta\pi\cdot\left(\frac{T_d}{T_0}\right)$$

For cases when damping is small, $T_d = T_0$

$\therefore \lambda = \pi\zeta$.

$$\therefore \lambda = \pi\cdot\frac{D}{D_0} = \frac{\pi\cdot D}{2\sqrt{KJ}}$$

Hence from (1), we get

$$\theta_1 = AQe^{-\lambda/2}$$

$$\theta_2 = AQ e^{-3\lambda/2}$$

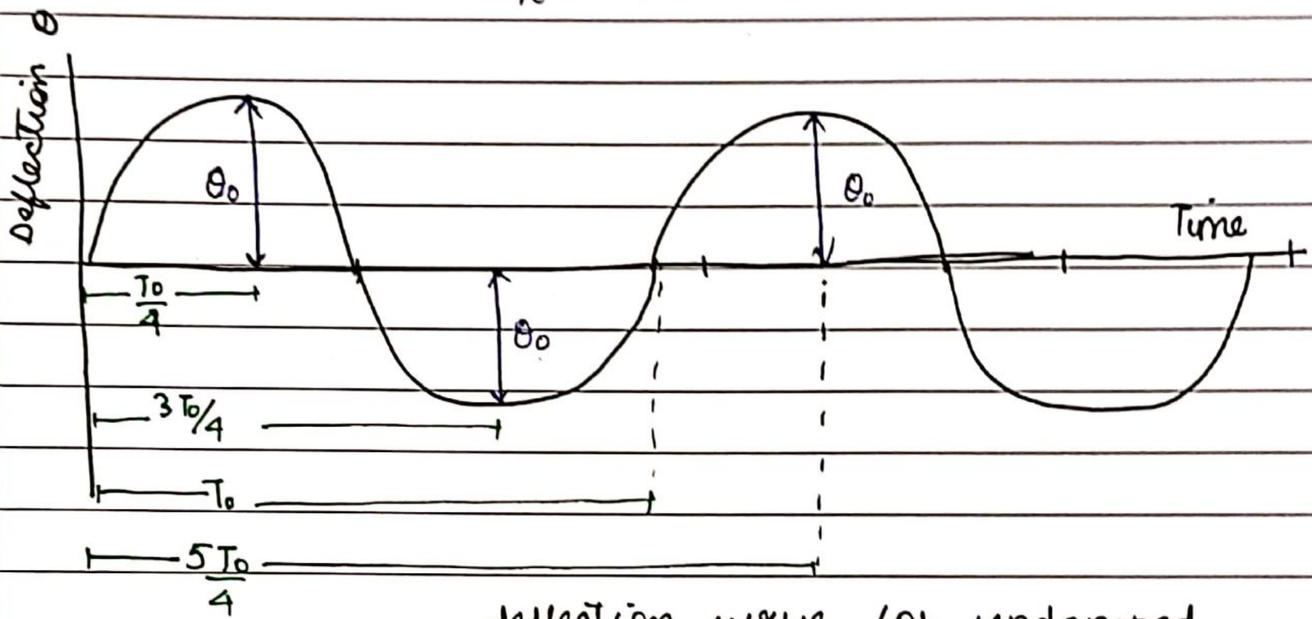
$\vdots$

$$\theta_n = AQ e^{-(2n-1)\lambda/2}$$

$$\theta = A.Q \sin\left(\frac{2\pi}{T_0}\right)\cdot t \qquad \left[\because D = 0 \quad e^{-\lambda/2J} = 1\right].$$

Amplitude of swings of undamped oscillations is

$$\theta_0 = AQ \sin \frac{2\pi}{T_0}\cdot\frac{T_0}{4} = AQ.$$



deflection curve for undamped galvanometer.

$$\theta_1 = \theta_0 e^{-\lambda/2}$$

$$\theta_2 = \theta_0 e^{-3\lambda/2}$$

$$\vdots$$

$$\theta_n = \theta_0^{-(2n-1)\lambda/2}.$$

Logarithmic decrement is logarithm of ratio of successive swings.

$\therefore$ logarithmic decrement $\lambda = \log\left(\frac{\theta_1}{\theta_2}\right)$.

Citizen

Topic Date

Ratio of successive swings is $\frac{\theta_1}{\theta_2} = \frac{e^{-\lambda/2}}{e^{-3\lambda/2}} = e^{\lambda}$

llly $\frac{\theta_2}{\theta_3} = e^{\lambda}$

$\therefore \frac{\theta_{n-1}}{\theta_n} = e^{\lambda}$

$\therefore \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} \times \frac{\theta_3}{\theta_4} \times \cdots \frac{\theta_{n-1}}{\theta_n} = e^{(n-1)\lambda}$

$\frac{\theta_1}{\theta_n} = e^{(n-1)\lambda}$

$\therefore \lambda = \frac{1}{n-1} \log_e \left(\frac{\theta_1}{\theta_n}\right)$

Again, $\theta_1 = \theta_0 e^{-\lambda/2}$ $\therefore \theta_0 = \theta_1 e^{\lambda/2}$

$\theta_0 = \theta_1 \left(1 + \frac{\lambda}{2}\right)$ approx.

But $\theta_0 = AQ$. Hence $AQ = \theta_1 (1 + \lambda/2)$

$\therefore$ Charge $Q = \frac{\theta_1}{A}\left(1 + \lambda/2\right)$

Substituting the value ; $A = \frac{G}{J}\sqrt{\frac{J}{k}}$

Charge $Q = \frac{J}{G}\sqrt{\frac{J}{K}}\ \theta_1 \left(1 + \frac{\lambda}{2}\right)$

Now, $T_0 = 2\pi\sqrt{\frac{J}{K}}$

$\therefore$ Charge $Q = \frac{J}{G}\sqrt{\frac{K}{J}} \cdot \frac{T_0}{2\pi\sqrt{J/K}} \cdot \theta_1\left(1+\frac{\lambda}{2}\right)$

$= \frac{K}{G} \cdot \frac{T_0}{2\pi} \, \theta_1\left(1+\lambda/2\right)$

Now we have to eliminate q.ty K and G.

Suppose a steady current I_g passing through galvanometer produces a steady deflection θ.

$\therefore \; G I_g = K\theta$ or $\frac{K}{G} = \frac{I_g}{\theta}$

Hence $Q = \frac{I_g}{\theta} \cdot \frac{T_0}{2\pi}\left(1+\frac{\lambda}{2}\right) \cdot \theta_1$

$= K_g \cdot \theta_1$

Where $K_g = \frac{I_g}{\theta} \cdot \frac{T_0}{2\pi}\left(1+\lambda/2\right)$ = constant of galvanometer.

Units of K_g = columb per radian.

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