Digital Signal Processing

1. Dig. Signal Processing - A computer based approach - S.K mitra TMH 2nd Edition
2. Discrete time signal processing - Oppenheim & Schaffer - PHI
3. Dig. Signal Processing - A practical approach - (2nd Edition - Pearson education Asia) Emmanuel C. Ifeachor & Barrie W Jervis
4. Digital Signal Processing - System Analysis and Design (Cambridge University Press) Paulo S.R Diniz, Eduardo A.B de Silva and Sergio L. netto

Signal :-

a variable i.e dependent on some independent variable (maybe funcn. of time or space or both)

eg.. speed of motor, torque of a motor (varying with time),
- flow of a liquid in a chemical process
- intensity of light (like in images, illumination level)
- " " mechanical vibrations.
- pressure
- current
- voltage, etc.

All these signals can be converted into electrical signals through transducers

- In HV systems for measurement of various parameters

PT → step down voltage
CT → " " current

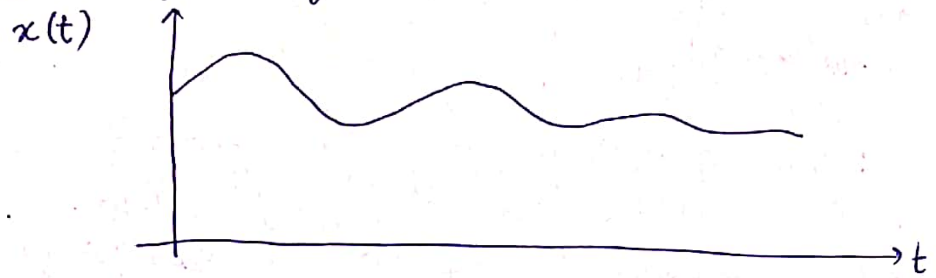
We use these signals for communication b/w
- man ⟷ man
- man ⟷ machine
- machine ⟷ machine

man to man communication need not always be recordable communication.
We communicate through our sense organs (eyes, gestures (Human speech)

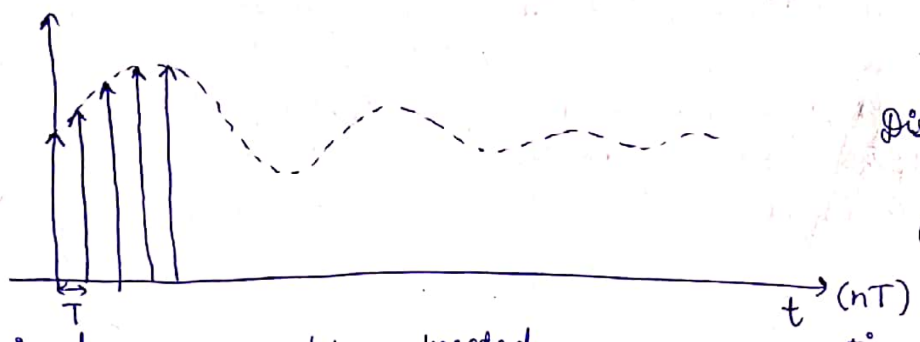
man ⟷ machine (interaction b/w the 2, oral command)

m/c ⟷ m/c (signals through computers eg. settelite communication)

Types of Signals:-

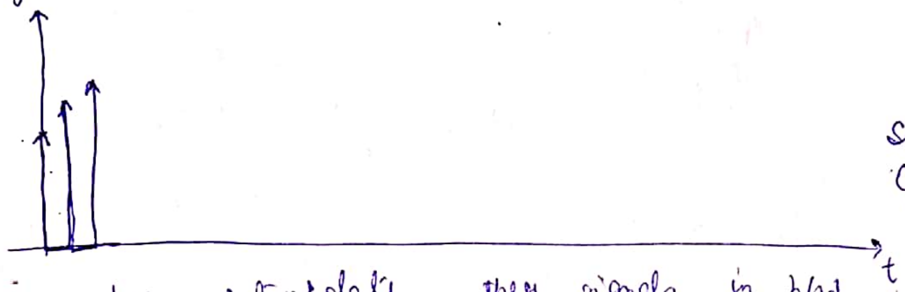


Continous signal



time
Discrete, signal
(nothing in b/w T
not defined)
(discrete only in time domain)

This signal can be treated as a continous signal as



Scaled delta functions
(zero in b/w)

One may also interpolate these signals in b/w

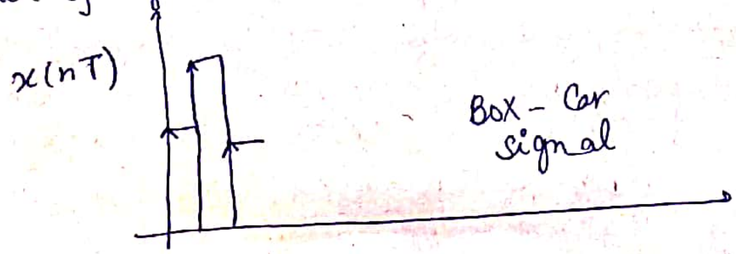
(see in filters)

If one is recording a discrete time signal through a digital ~~signal~~ device, then there is a limitation (concept of resolution) limited memory size.

↑ say this value is 5.625 units
if only decimal storage capacity then it'll be 5.6 (truncation) or 5.7 (round-off) → not beyond one decimal
so, only certain discrete steps at intervals of 0.1,...

(nothing in b/w)
discrete in time domain as well as in magnitude
↳ These are called **Digital signal**

extension of digital signal



Box-Car signal

amplitudes recorded & assumed to remain const. till next signal appears
so, a continous time signal but at definite levels
(Sample & Hold ckt-)

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$x(t) \rightarrow$ only time dependent

In discrete time domain,

$x_a(nT) \rightarrow x[n]$ array or sequence of data (Sampling time is not changing)

Variable in space

$S(x, y, z, t)$ Seismic studies

eg: colour TV $\rightarrow$ 3 primary colours

s_r, s_g, s_b

Each pixel consists of 3 elements (R, G, B). Each comp. will've different levels of intensity

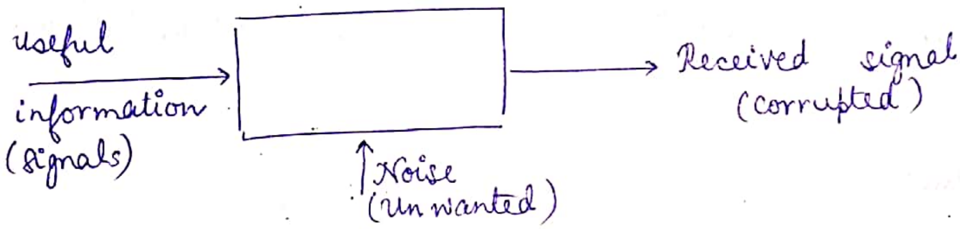
Each element will depend on 2 element ($x, y \rightarrow$ 2D)

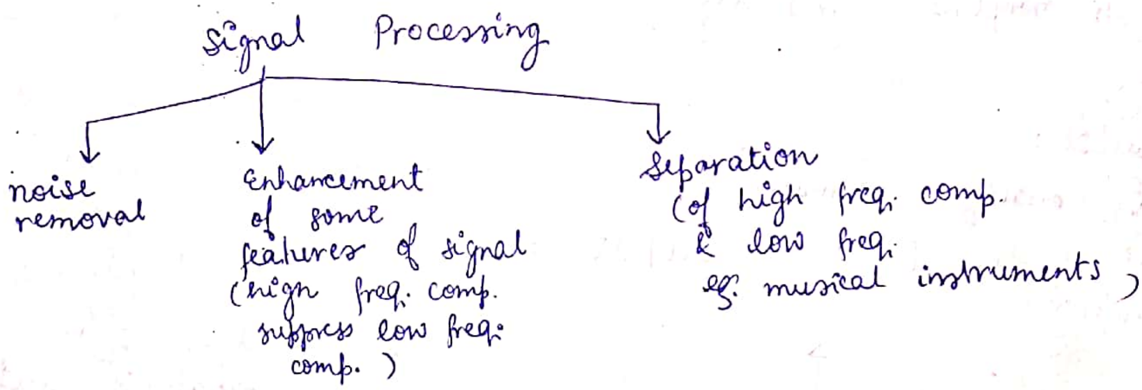
$S_r(x, y, t)$ (how picture is changing with time

$S_g(x, y, t)$

$S_g(x, y, t)$

In signal processing :-





<u>Field of Application</u> :-

1. Acoustics, vibration, speech, data communication, SONAR/RADAR,
2. Seismology - oil exploration, earthquake.
3. Bio-medical — EEG, ECG
4. Robotics
5. Instrumentation & control
6. Image processing -
7. Consumer electronics

Advantages of DSP :-

① For fast processing of data (parallel process architechure, super computers)

② Exact reproduction or repeatability (conducting same exp. at diff. time we'll get same result → no aging effect ; with analog component (physical comp.) there is always chance of noise creeping in)

③ Easy parameter adjustment (always change a parameter / constant); not so easy for analog system → flexible sys.

④ Guaranteed accuracy (determined by no. of bits

⑤ Small size and low cost (with advance of VLSI)

Disadvantages

① Design time - Speed and cost for large B.W signal

② Finite word length (errors keep on accumulating)

Signals :- come from sensors

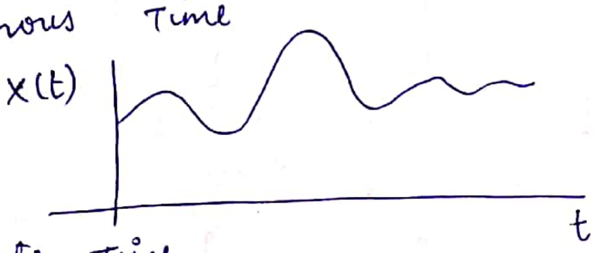
Systems :- process signals to produce other signals

Gas Pedal → $\boxed{\text{Car}}$ → Velocity (that drives wheels)

CD (Digital) → $\boxed{\text{stereo}}$ → Sound (cont.)

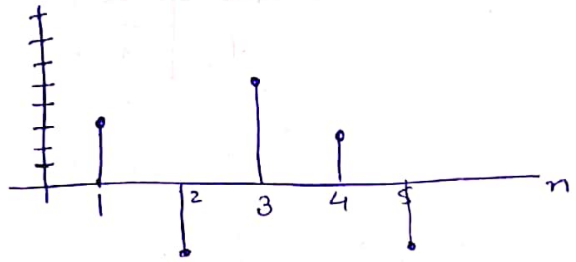
continous Time

x(t)



Discrete Time

x[n]



x[1.5] ⇒ not defined

- D/A conv.
- ∞ resolution on y-axis not possible → Called Quantization

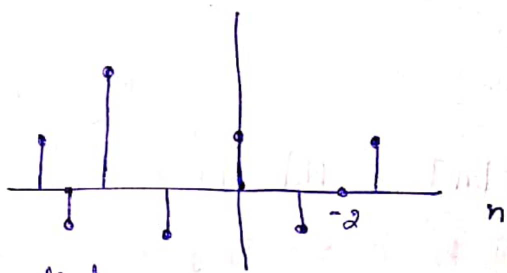
Basic operations on signal

① Flipping

X[n]

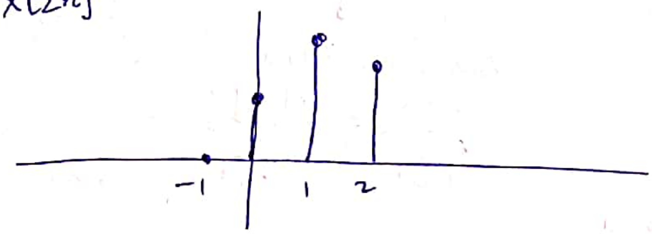
→

x[-n]



② Scaling (playing audio signal twice as fast overall duration of signal decreases by 2)

x[2n]

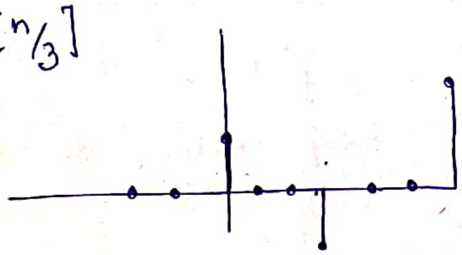


we're losing information (thrown away some samples)

In cont. signal no loss

Samples are lost

X[n/3]



slowing down a signal by a factor of 3

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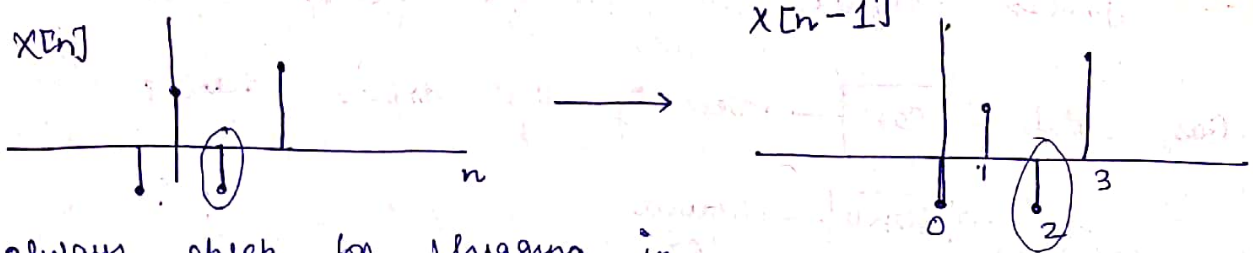
③ shifting

$x[n-n_0]$

now am I delaying the signal (1 unit late)



always check by plugging in

$Y[n] = x[n-1]$

$Y[2] = x[1]$

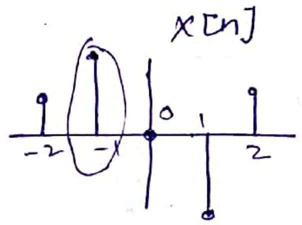
$x[-2n+3]$

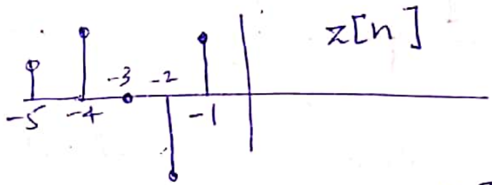
Order to follow:-

Shift, Flip, Scale

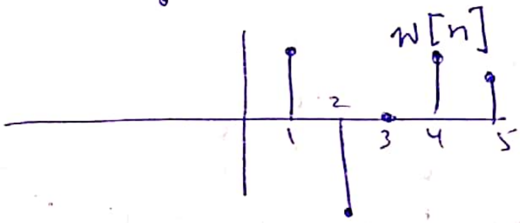
Define a new signal

$z[n] = x[n+3]$





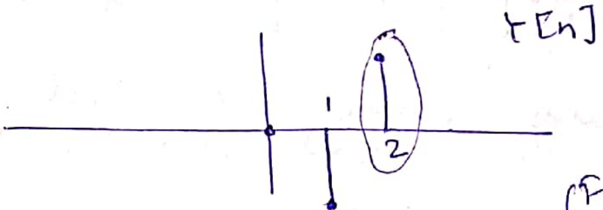
$w[n] = z[-n]$



$Y[n] = w[2n]$

$z[-2n]$

$x[-2n+3]$



(Fill zero!)

One can check

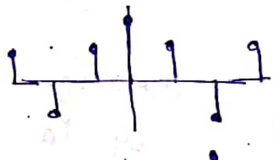
$Y[2] = x[-1]$

$Y[1] = x[1]$

Properties of signals:-

① even

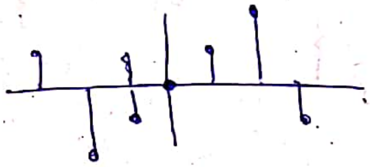
$x[n] = x[-n]$



② odd

$x[n] = -x[-n]$

$x[0] = -x[0] = 0$



Every signal has an even part and an odd part.

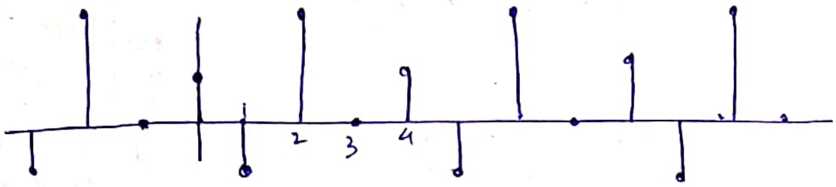
$$Ev[X[n]] = \frac{1}{2}(X[n] + X[-n])$$

$$Od(X[n]) = \frac{1}{2}(X[n] - X[-n])$$

show that these are even & odd.

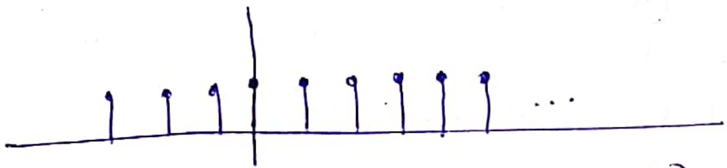
② Periodicity

$X[n] = X[n+N]$ i.e signal repeats itself after a certain no. of integer slips



Period $N = 4$

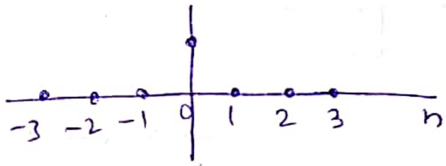
A const. signal is also periodic its period is $\underline{N=1}$



Some functions (special signals)

① Delta function (unit sample)

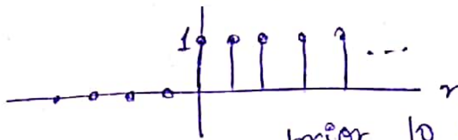
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



② unit step function

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

u[n]



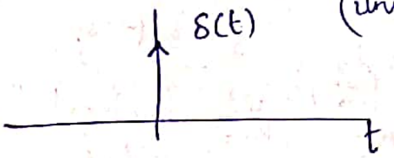
prior to zero its off & at 0 it turns ON (label ur axis!)

$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

(Taking apart the funcn.) If $k = -\infty$ to ∞ we've a const. signal.

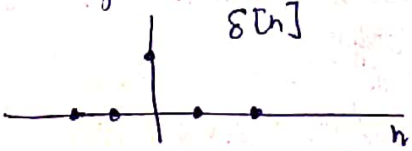
In cont. time, delta funcn. (unit amount of energy)



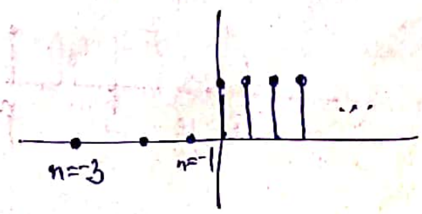
$$\int_{\tau=-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

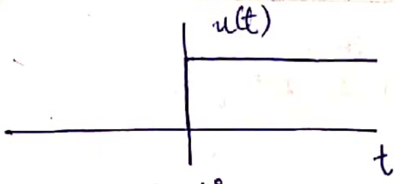
Analogy b/w integration & running sum

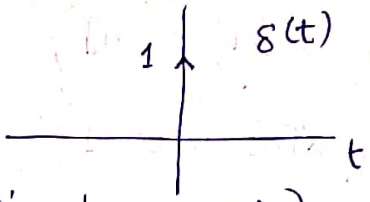
Same for delta funcn. $\delta[n]$



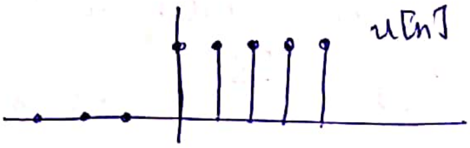
$$u[n] = \sum_{k=-\infty}^{\infty} \delta[k]$$

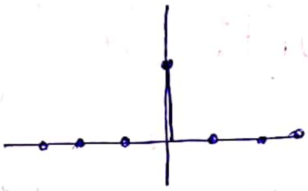


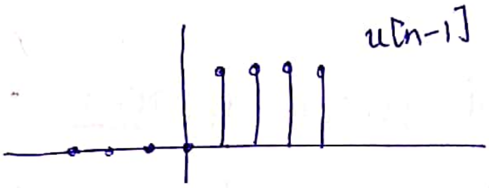




Derivative
(lech. at 0 derivate not defined; that's why arrow)





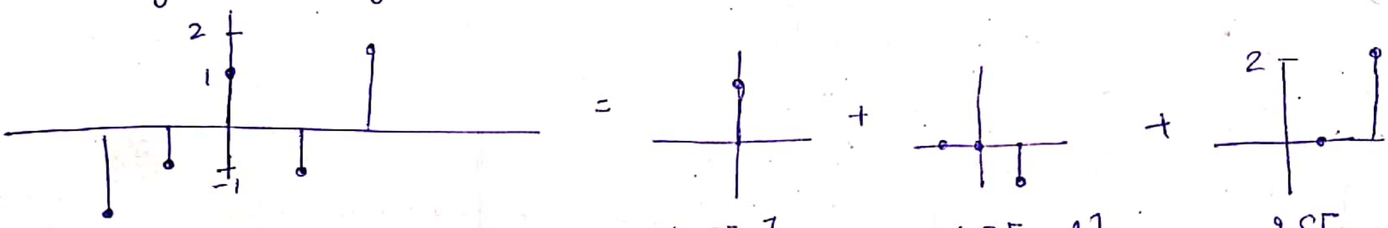


Derivative
difference
(made smaller & smaller)

$\delta[n] = u[n] - u[n-1]$

$\approx \delta(t) = \frac{du(t)}{dt}$ (in cont. time)

- making a signal out of delta funcn.



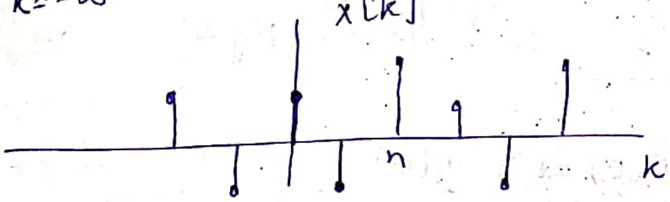
$1 \cdot \delta[n]$ $\quad -1\,\delta[n-1]$ $\quad 2\,\delta[n-2]$

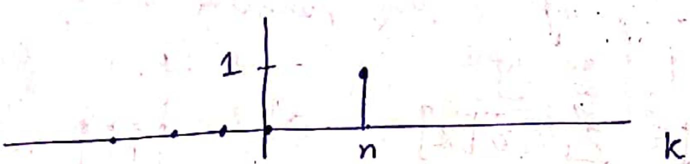
$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}$$

At every value of k I multiply the δ funcn. by corr. value of x and add.

- sampling property

$$\sum_{k=-\infty}^{\infty} x[k]\,\delta[k-n] = x[n]$$





multiplying a lot of zero
only time we get a non-zero value is when δ funcn. is firing up.

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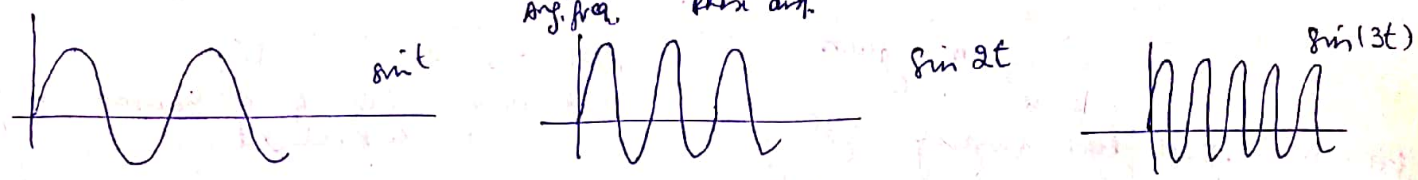
In cont. time sinusoids, we had the concept of low frequency & high frequency

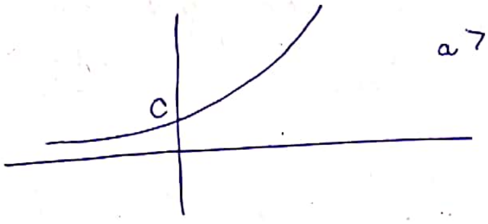
$$x(t) = A \sin(\underset{\text{Ang. freq.}}{\omega_0} t + \underset{\text{phase diff.}}{\phi})$$



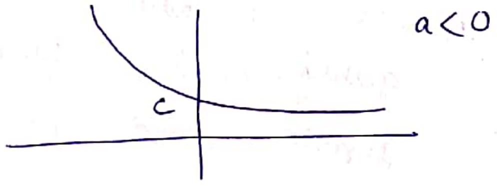
low freq. ⟶ high freq.

• exponentials :-

$x(t) = Ce^{at}$ C, a real



speeding up or slowing down



when C and a are complex

$$x(t) = Ce^{at}$$

$C = |C|e^{j\theta}$ and $a = r + j\omega_0$

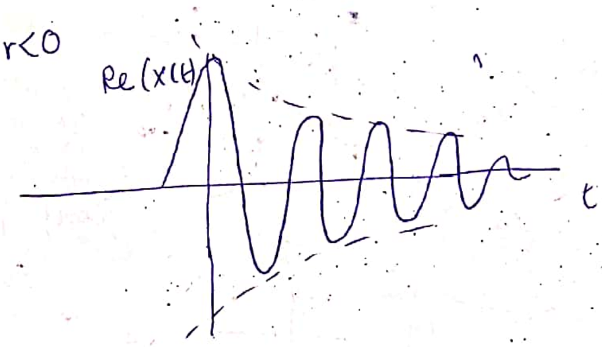
$$x(t) = |C| e^{j\theta} e^{(r+j\omega_0)t}$$

$$= |C| e^{rt} e^{j(\omega_0 t + \theta)} \longrightarrow \text{(A circle with radius)}$$

Real part & imag. part

$$= |C| e^{rt} [\cos(\omega_0 t + \theta) + j \sin(\omega_0 t + \theta)]$$

If $r < 0$



Expanding or contracting cosine envelope

sense of period in this relate a freq. to complex exponentials.

In a similar way

$$x[n] = C\alpha^n = Ce^{\beta n}, \quad \alpha = e^{\beta}$$

$$= |C||\alpha|^n (\cos(\omega_0 n + \phi) + j \sin(\omega_0 n + \phi))$$

In C.T sines & cosine just keep on ↑ in freq. but that is not true in D.T

D.T sinusoids are different

$e^{j\omega_0 n}$ what is $e^{j(\omega_0+2\pi)n}$?

In cont. time it would be faster freq. sinusoid with incres. of 2π in freq.

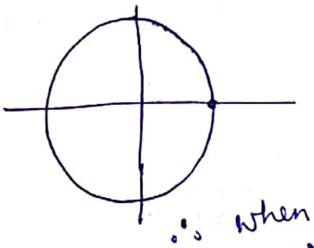
$= e^{j\omega_0 n + j2\pi n}$

$= \cancel{e^{j2\pi n}}^{1} \cdot e^{j\omega_0 n}$

note n is an integer and ω_0 is freq. of CT sinusoid which is discretized.

For $n=0$, that's like saying e^{j0}

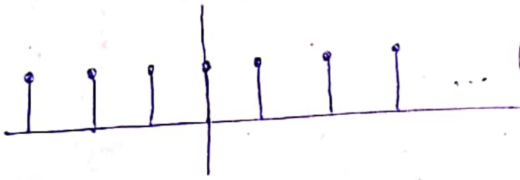
- If $n=1$ one full circle
- If $n=2$ $e^{j4\pi}$ twice around the circle



∴ when I add 2π to any freq. we end up getting the same freq. → no ∞ high freq. in Discrete world we can only go about a circle

There's only a 2π wide range of frequencies in D.T

For $\omega=0$; $e^{0n}=1$ const

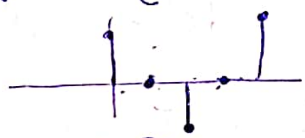


T=1
f=1

Periodicity is in variable n not in freq. ω

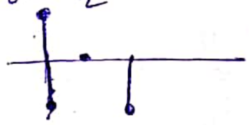
lowest frequency

$\omega_0=\pi/2$ $e^{j\pi/2 n}$. {Re}

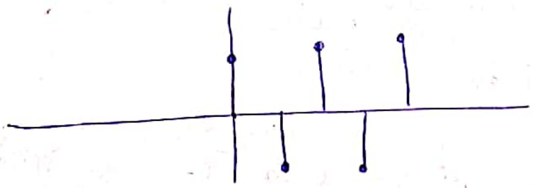


$T=4$
$f=1/4$

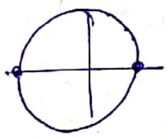
$\omega_0=3\pi/2$



highest freq. $\omega=\pi$

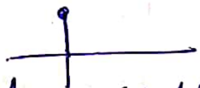


$e^{\pi n} = \begin{cases} 1 & n=\text{even} \\ -1 & n=\text{odd} \end{cases}$



signal is bouncing back & forth in comp-plane

If I take real part of the complex expon. $\cos(\omega_0 n)$ if I'm increasing the freq. in reality I'm increasing the phase i.e $\cos(\omega_0 n + kn)$ & the max. freq. change would corr. to a max. phase change which is π in this case rather than lowing

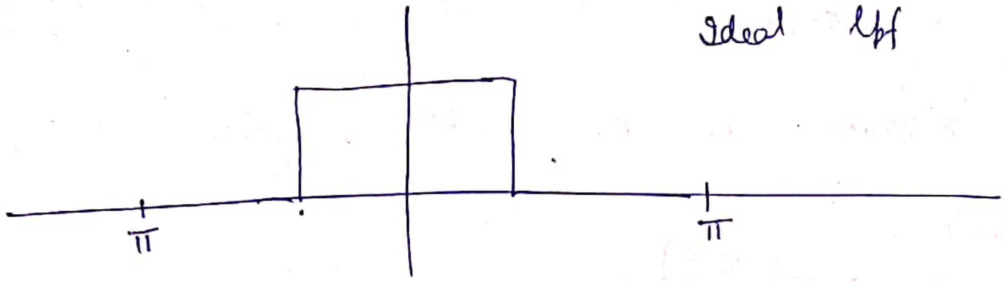


In what freq. can I have max phase sep. b/w sample $n=0$ & $n=1$

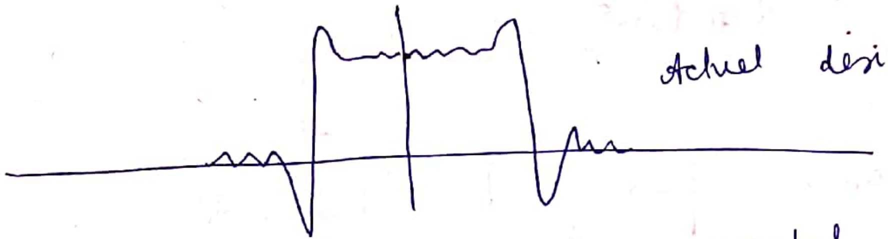
~~At~~ At $\omega_0=\pi+\Delta\omega$

$e^{j(\pi+\Delta\omega)n} = e^{j(\pi+\Delta\omega-2\pi)n} = e^{j(\Delta\omega-\pi)n} = e^{-j(\pi-\Delta\omega)n}$

(To design a lpf ,
Discrete Time , Digital l.p.f



Ideal lpf



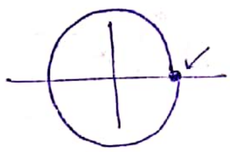
Actual design filter

Lastly one needs to be careful whether a signal is periodic in discrete world i.e not all cosine are periodic in D.T !

when is $e^{j\omega_0 n}$ periodic (discrete) ?

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} \quad \text{For some integer } N$$

$$e^{j\omega_0 n} \cdot \underbrace{\boxed{e^{j\omega_0 N}}}_{1} = e$$



I should be here for $e^{j\omega_0 N} = 1$

i.e $\omega_0 N = 2\pi K$ (some even multiple of π) for integer K

$\therefore$ freq., $\omega_0 = \dfrac{2\pi K}{N}$

or period $N = \dfrac{2\pi K}{\omega_0}$ integer

eg:

$$x[n] = \cos\left[\frac{4\pi}{5} n\right]$$

$\therefore N = \dfrac{2\pi K}{4\pi/5}$ must be integer

$$= \frac{10\pi K}{4\pi} = \frac{5}{2} K$$

$\therefore K = 2 , N = 5$

This signal is periodic with period 5 cr.(?)

$X[n] = \cos(7n)$

$N = \frac{2\pi k}{7}$ no integer k which will make N an integer

② Systems → process signals to create other signals

System

$p(t) \longrightarrow \boxed{\text{Car}} \longrightarrow v(t)$

Pressure on gas pedal

velocity

$o[n] \longrightarrow \boxed{\text{Economy}} \longrightarrow g[n]$

Price of Barrel of oil on Day n

Daily price of gallon of gas on day $[n]$

Representation

System

$X[n] \longrightarrow \boxed{H} \longrightarrow Y[n]$

$X[n] \longrightarrow Y[n]$

$Y[n] = H(\cancel{H}X[n])$

eg. $Y[n] = \frac{1}{5}\sum_{k=-2}^{2} X[n-k]$

$= \frac{1}{5}\left[x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2] \right)$

special case of Moving Avg. Filter (Exp.)

In C.T a lot of systems are described by Diff. eqn.

$y'(t) + a y(t) = b x(t)$

In D.T diff. eqn.

$y[n] + a\, y[n-1] = b\, x[n]$

learn how to solve systems when an I/P is given what is the O/P. machinery of DSP makes solving diff. eqn. simple (Z transform)

Difference eqn. arise from electrical and mechanical problems

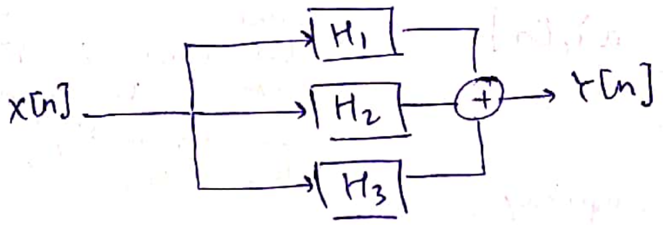
Connecting Systems

① $X[n] \longrightarrow \boxed{H_1} \longrightarrow \boxed{H_2} \longrightarrow Y[n]$

Serial / Cascade

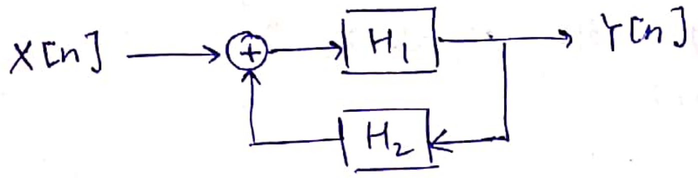
eg. Stereo (pre-amp, equalizer)

Scanned by CamScanner

② 

Parallel

signal that goes through multiple systems & combine these to get O/P signal (graph. eq.)

③ 

Feedback

eg. cruise control of car
I/P → Pressure on gas pedal
O/P → for const. speed

System Properties:- * memoryless system → O/P $y[n]$ at every value of n depends only on I/P $x[n]$ at the same value of n eg. $y[n] = f(x[n])^2$

① Causality :- A system is causal if the O/P at time n only depends on the I/P up to time n

eg. $Y[n] = x[n] - 2x[n-1]$ ✓

$Y[n] = x[n+3]$ ✗ not causal

check if some future time appears for the system

Is every real world system causal?

It depends on the way we think about. eg.

car is causal (It doesn't know how you're gonna steer / break a minute in future)

* a graph equalizer is causal (If signals are stored in memory and at a later time they're used by a system then such signals are treated / as advanced 1-D signal / or future signal because such signals are already present before the system has started its operation

/ ~~image processing → not causal~~

eg. Population growth
weather forecasting

② Linearity

2 properties that go with linearity

~~$x_1[n] + x_2[n] \to$~~ $x_1[n] \to Y_1[n]$

$x_2[n] \to Y_2[n]$

$x_1[n] + x_2[n] \longrightarrow \boxed{H} \longrightarrow Y_1[n] + Y_2[n]$ $\boxed{\text{Additivity}}$

$a X_1[n] \longrightarrow \boxed{H} \longrightarrow a Y_1[n]$

Homogenity

multiply one of the signal by a const. the corr. O/P is also multiplied by that const.

combining these 2 properties together

$a_1 X_1[n] + b X_2[n] \longrightarrow \boxed{H} \longrightarrow a Y_1[n] + b y_2[n]$

- to show is true for any I/P signal X_1, X_2
- system is prop. on I/P (linearization)

eg. $Y[n] = X[n] - 2X[n-1]$

$X_1[n] \longrightarrow X_1[n] - 2X_1[n-1] = Y_1[n]$

$X_2[n] \longrightarrow X_2[n] - 2X_2[n-1] = Y_2[n]$

what is the response to $z[n] = X_1[n] + X_2[n]$?

(be methodical)

$z[n] \longrightarrow \boxed{H[n]} \longrightarrow z[n] - 2z[n-1]$

$= X_1[n] + X_2[n] - 2X_1[n-1] - 2X_2[n-1]$

$= (X_1[n] - 2X_1[n-1]) + (X_2[n] - 2X_2[n-1])$

$= Y_1[n] + Y_2[n]$ Additivity ✓

now, lets consider

$z[n] = aX_1[n]$ — $z[n] \longrightarrow \boxed{H} \longrightarrow z[n] - 2z[n-1]$

$= a(X_1[n] - 2X[n-1])$

$= a Y_1[n]$ Homogenity ✓

little bit time consuming to do these step.

better to do all at once

$z[n] = a X_1[n] + b X_2[n]$

$z[n] \longrightarrow \boxed{H} \longrightarrow z[n] - 2z[n-1]$

$= a X_1[n] + b X_2[n] - 2(a X_1[n-1] + b X_2[n-1])$

$= a(X_1[n] - 2X_1[n-1]) + b(X_2[n] - 2X_2[n-1])$

$= a Y_1[n] + b Y_2[n]$

Linear ✓

Some systems may obey one but not the other

It is not ~~does not~~ that mean one implies the other.

In theory we've to check both for linearity

(Guess whether a system is linear → contains instances of signal of previous I/P.)

eg. $y[n] = 3x[n] + 5$ linear or non L.?

$x_1[n] \rightarrow 3x_1[n] + 5 = y_1[n]$

$x_2[n] \rightarrow 3x_2[n] + 5 = y_2[n]$

Let $z[n] = x_1[n] + x_2[n]$

$z[n] \rightarrow 3z[n] + 5$

$= \underline{3(x_1[n] + x_2[n]) + 5}$ → Is this same as $y_1[n] + y_2[n]$

not linear (if it fails for one it fails for all)

usually proved with constant funcn. or δ funcn)

counter example

Let $x_1[n] = 1$ $x_2[n] = 2$

$y_1[n] = 3 + 5 = 8$ $y_2[n] = 11$ which should be $2y_1 = 16$

- Homogenity

$a\,x_1[n] \longrightarrow a\,y_1[n]$

for any const.

∴ A linear system must have $0 \rightarrow 0$

(In above eg. $0 \rightarrow 5$)

③ Time Invariance :-

system behaves the same way regardless of 'when' I/P is applied

$x[n] \longrightarrow \boxed{} \longrightarrow y[n]$

$x[n-n_0] \longrightarrow \boxed{} \longrightarrow y[n-n_0]$

put toast at 3pm comes out 3:05

eg. $y[n] = x[n] - 2x[n-1]$

Time invariant?

Dummy signal $z[n] = x[n-n_0]$

$z[n] \longrightarrow \boxed{} \longrightarrow z[n] - 2z[n-1]$

$= x[n-n_0] - 2x[n-1-n_0]$

Is this ?? $y[n-n_0]$

$= x[n-n_0] - 2x[n-n_0-1]$

$= x[n-n_0] - 2x[n-1-n_0]$ ✓

eg. consider $y[n] = x[n^2]$ $\qquad x[n] \longrightarrow \boxed{} \longrightarrow x[n^2]$

Let $z[n] = x[n-n_0]$

$z[n] \longrightarrow \boxed{} \longrightarrow z[n^2]$

$x[n^2-n_0]$

$\overset{?}{=} y[n-n_0] = x[(n-n_0)^2] = x[n^2 - 2nn_0 - n_0^2]$ X

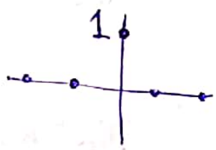
not a time invariant system

Counter example to Time Invariance:

Try δ- funcn.

$y[n] = x[n^2]$

Response to $x[n] = \delta[n]$

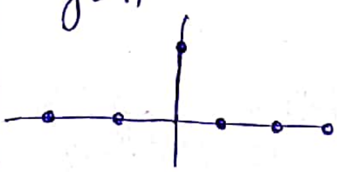


$y[0] = x[0]$

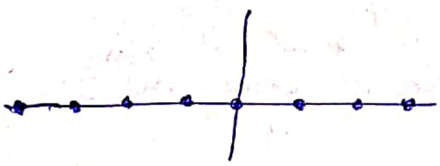
$y[1] = y[-1] = x[1]$

$y[2] = y[-2] = x[4]$

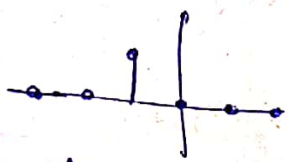
$y[n]$



Response to shifted $x[n] = \delta[n+1]$



not time invariant



(Impulse response is useful in LTI systems)

Hi .

Linear, Time Invariant systems (LTI systems) (assume all as LTI)

Real world systems are often modelled as LTI

ⓐ often a good approximation

ⓑ analysis is very easy / powerful

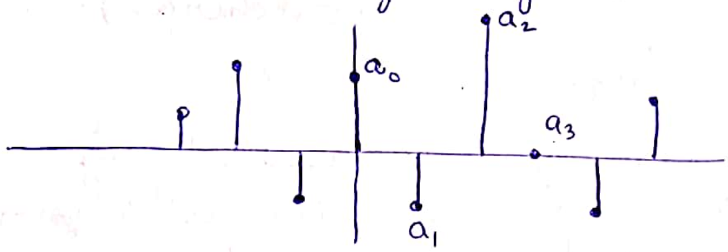
Key concept :: is Superposition for LTI systems

Let

$$a_1 x[n] + a_2 x[n-1] + a_2 x[n-2] + \dots \longrightarrow \boxed{LTI}$$

$$\Rightarrow a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \dots$$

— consider an arbitrary signal



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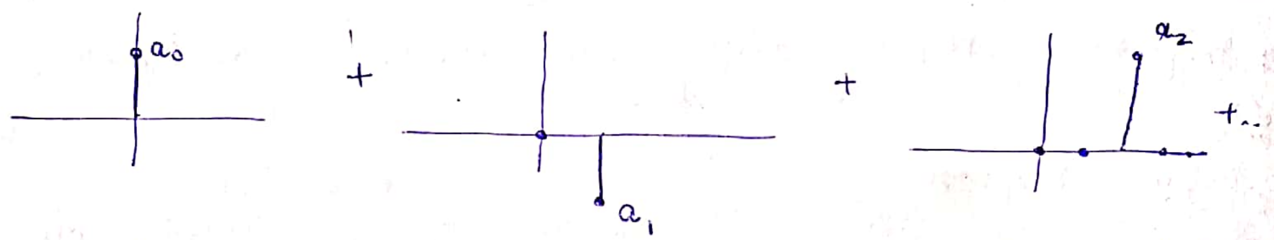

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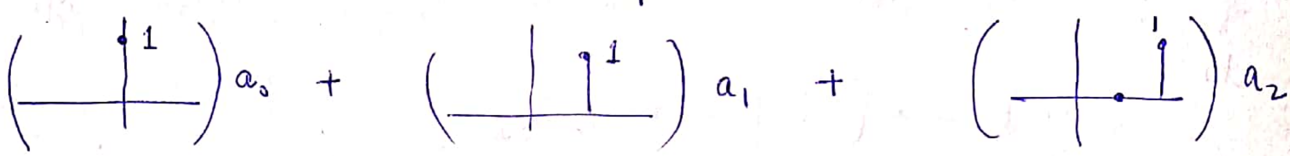

$$= a_0 \delta[n] + a_1 \delta[n-1] + a_2 \delta[n-2] + \dots$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \underline{\delta[n-k]}$$

⟶ This δ func. fires only at $n=k$ & when it fires its value is whatever this value $x[k]$ is

"Sifting" property of δ - function

Lets suppose we've an LTI system

$$\delta[n] \longrightarrow \boxed{H} \longrightarrow h[n]$$

Unit Impulse (Delta funcn.) — Impulse Response

— 'I can figure out what is the response of the system to any input x by only looking at what the system did to the impulse what is the response to $x[n]$?

$$H(x[n]) = H\left(\sum_{k=-\infty}^{\infty} x[k]\ \delta[n-k]\right)$$

$x[k]$ are just no.'s multiplying the heights of δ-func (const.) $\delta[n]$ is taken as i/p

By linearity

$$= \sum_{k=-\infty}^{\infty} x[k]\ H(\delta[n-k])$$

By time invariance, (shifted impulse resp.)

$$= \sum_{k=-\infty}^{\infty} x[k]\ h[n-k]$$

$$\underset{\text{o/p}}{y[n]} = \underset{\text{i/p}}{x[n]} * \underset{\text{impulse resp.}}{h[n]} \qquad \text{(convolution)}$$

[A LTI sys can be entirely characterized by what it does to the delta func. (no need to form a table this is i/p this is o/p)]

→ Impulse resp. fully characterizes the system