Impulse Response

last time, If I knew what the response of a linear sys. was to

$$\delta[n] \longrightarrow \boxed{H} \longrightarrow h[n]$$

unit impulse — Impulse Response

& I want to know what'll happen if I put an arbitrary signal

LTI

$$x[n] \longrightarrow \boxed{H} \longrightarrow y[n]$$

We showed that this O/P was convolution of I/P with impulse resp.

$$y[n] = x[n] * h[n]$$

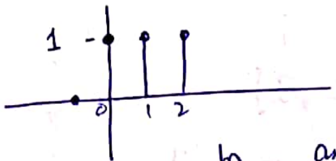
$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

What does this convolution sum doing?

Consider this LTI system

The HOD. $y[n] = x[n] - 2x[n-1] + 3x[n-2]$

what is the response to

$x[n] =$  ? $\Rightarrow [\underset{\uparrow}{1}, 1, 1]$ origin $(n=0)$

there're many ways to answer this

① **Direct**

$y[-1] = 0$ Also $-2, -3, \ldots$

$y[0] = 1$

$y[1] = 1 - 2(1) = -1$ $[\underset{\uparrow}{1}, -1, 2, 1, 3]$ $n=0$

$y[2] = 1 - 2 + 3 = 2$

$y[3] = -2 + 3 = 1$

$y[4] = 3$

list of no. One thing that'll make this a bit easier is to represent as $\Rightarrow$

never do this thing in practice (large I/P)

② Using convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

what is the impulse response?

go back to formula & see what the I.R is

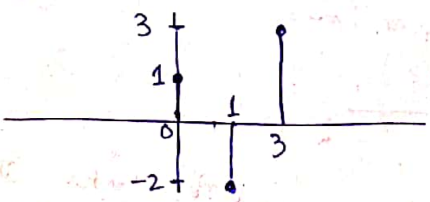
$\delta[n] = [1]$

$h[0] = 1$

$h[1] = 0 - 2 = -2$ $[\underset{\uparrow}{1}, -2, 3]$

$h[2] = 0 - 0 + 3 = 3$

$h[3] = 0$



mark the heights!

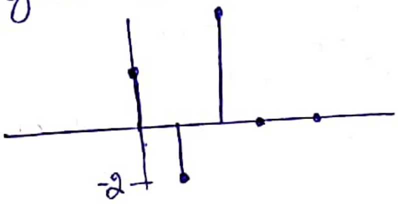
$y[n] = x[0]\, h[n] + x[1]\, h[n-1] + x[2]\, h[n-2]$

that's like saying I'm doing is I'm multiplying these delayed versions of I.R by the coeff. of I/P signal.

That's like

Original I.R

$x[0]\, h[n]$



$[\underset{\uparrow}{1}, -2, 3]$

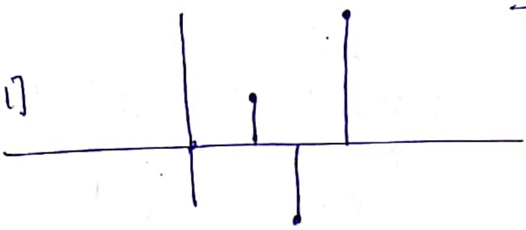
$[\underset{\uparrow}{0} \;\; 1 \;\; -2 \;\; 3]$

$[\underset{\uparrow}{0} \;\; 0 \;\; 1 \;\; -2 \;\; 3]$

$= [1, -1, 2, 1, 3]$

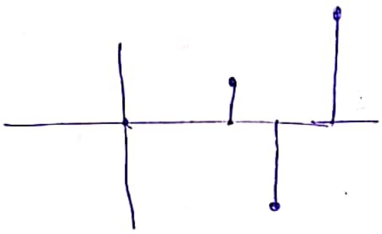
+

$h[n-1]$



+

$h[n-2]$



Add these up!

=

It's still tedious ; add up zillion copies of shifted response everytime we've new I/P signal

That leads to more graphical method of doing convolution

(3) Flip & slide one signal

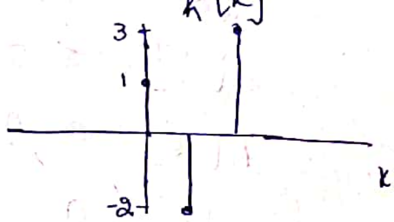
$$\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Another way of looking at this is following:-

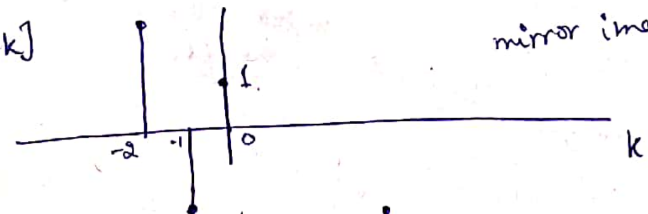
$x[k]$ is a funcn. of k

$x[k]$

$h[k]$

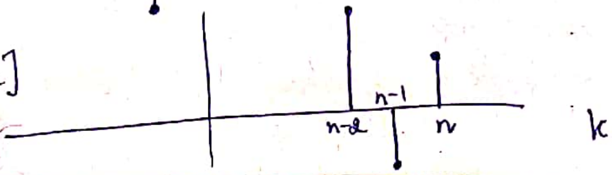


$h[-k]$



mirror image.

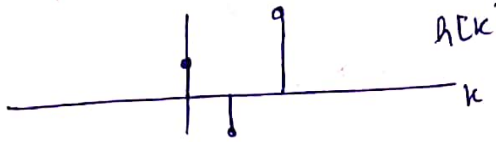
$h[n-k]$

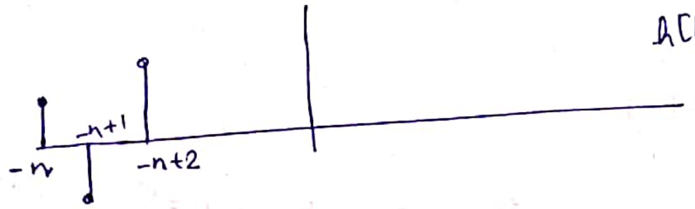


looks like the signal slid to the right by n units

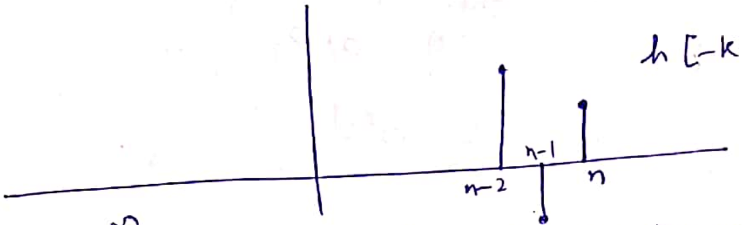
take what used to be at k=0 & move it over to n mul. corr. entries tog. & sum them.

$h[k] \longrightarrow h[\underset{\uparrow \text{const.}}{n}-k] = h[-k+n]$



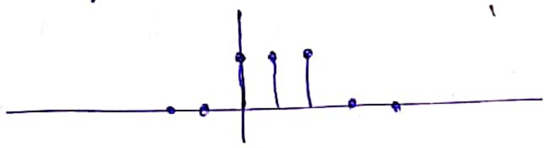


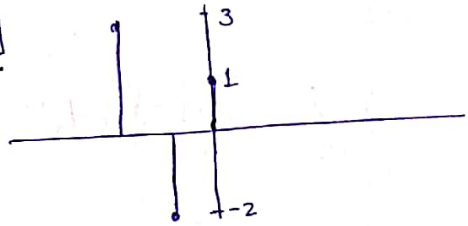
n is a +ve no.



If it is to be done in proper order!

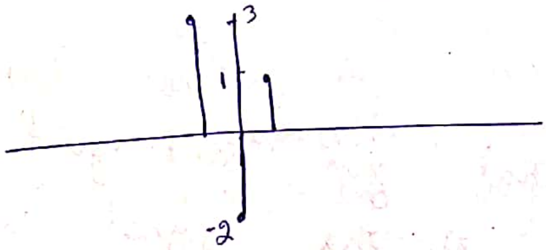
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]\, h[-k] \cancel{+ x[1] h[1-k]}$$





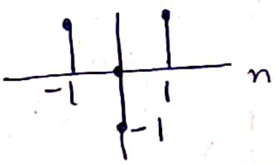
$y[0] = 1$

$$y[1] = \sum x[k]\, h[1-k]$$

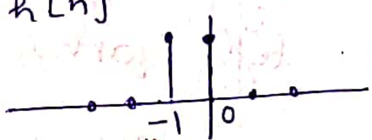


$y[1] = -2 + 1 = -1$

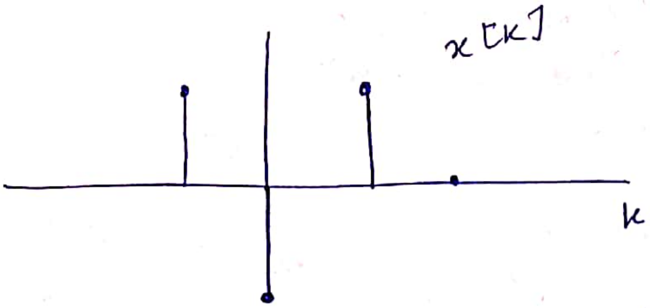
eg. 2 $x[n] =$



$h[n]$



1st thing I draw them as a function of k



This is for n=0 position of O/P

I look at the corr. products & add

$y[0] = 0 - 1 + 1 = 0$

when I move the signal to the right see the overlaps

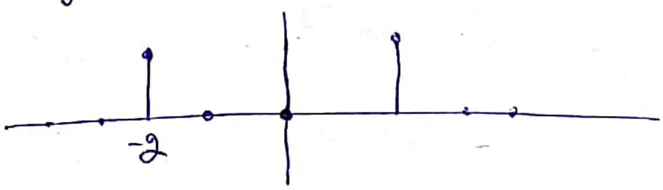
$y[1] = 1$

$y[2] = 0$

Also move the signal to the left $x[-1] \neq 0$

$y[-1] = 0$

$y[-2] = 1$



∴ It is important to know that you can't just stop for the right part of the signal but also for the left part if $x[k] \neq 0$ for $k < 0$

Faster way of doing convolution!

④ convolution Array (Wozny)

h \ x	1 ↓	1	1
→ 1	1	1	1
-2	-2	-2	-2
3	3	3	3

[1 -1 2 1 3]
↑

look at the diagonals of the table & add up the corr. values

- How do I know what is the zero element of the result

- mark the zero response of the I/P and zero resp. of Im. R. & where the 2 cross is the zero element of the O/P

eg ②

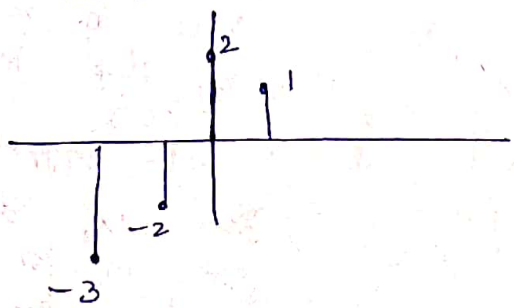
	1	-1 ↓	1
1	1	-1	1
→ 1	+1	-1 ↓	1

(3rd sum → see arrow)

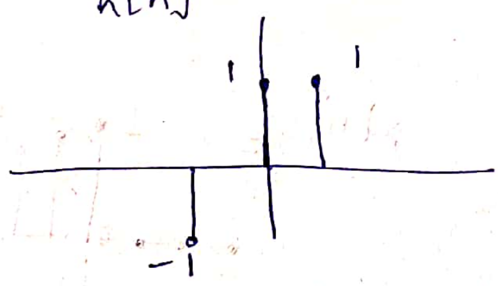
[1 0 0 1]
↑

- when you've a finitely I/P signal & impulse resp. then this is reasonable way to use this
→ but cannot for any I/P signals particularly those that're ∞ long.

eg ③ x[n]



h[n]



Scanned by CamScanner

	-3	-2	2	1	x
-1	3	2	-2	-1	
$\rightarrow 1$	-3	-2	$+2$	$+1$	
1	-3	-2	2	1	

$$[\,3 \quad -1 \quad -7 \quad -1 \quad 3 \quad 1\,]$$

(4th elements add')

• Its true that a lot of times we deal with what are called F.I.R systems (ones for which $h[n] \neq 0$ for finite no. of values) & often we're finite inputs also but a lot of times we're looking at I/Ps that're ∞ large.
So, in that case we still need to find out how to do convolution

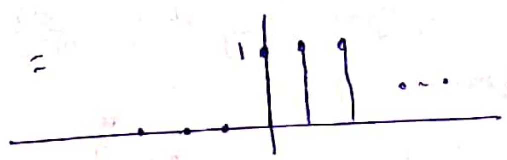
So,

eg. $x[n] = \alpha^n u[n] \qquad \alpha \in (0,1)$

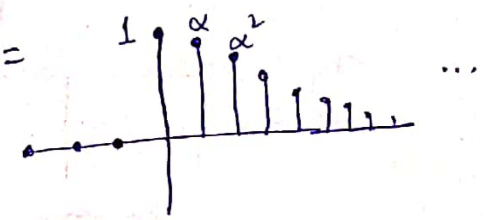
$h[n] = u[n]$

1st good to visualize what the signals look like

$h[n] =$



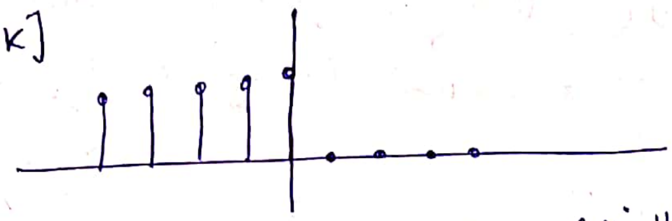
$x[n] =$



Since its multiplied by step funcn. (<0) nothing happens
This exp. decay's goes on for a long time for any value of n little non-zero x

• It doesn't matter which (order) signal I flip & shift the signal (either x or h)
In this case it'll be easier to choose h

$h[-k]$



• I can see for $n<0$ (shifting $h[-k]$ to left) I will be having nothing (no overlap) $y[-1] = 0$

• Right at zero

$$y[0] = 1 \times 1 = 1$$

$$y[1] = \text{shift one to right} = 1 \times 1 + \alpha$$

$$y[2] = 1 + \alpha + \alpha^2$$

$$y[n] = \begin{cases} 0 & n < 0 \\ \displaystyle\sum_{k=0}^{n} \alpha^k & n \geq 0 \end{cases} \qquad (\text{i.e. } n = 2 \quad 1+\alpha+\alpha^2)$$

$$= \underbrace{\left(\sum_{k=0}^{n} \alpha^k\right)}_{\text{g.p}} \cdot \underline{u[n]} \rightarrow \text{for } n<0 = 0$$

Remember

$k' = 0$
$n+1-(n+1)$
$k' = k - (n+1)$

• $$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha} \qquad 0 < \alpha < 1$$

$$(1+\alpha+\alpha^2+\dots)(1-\alpha) \stackrel{?}{=} \begin{array}{l} 1+\alpha+\alpha^2+\dots \\ \quad -\alpha-\alpha^2-\dots \end{array} = 1$$

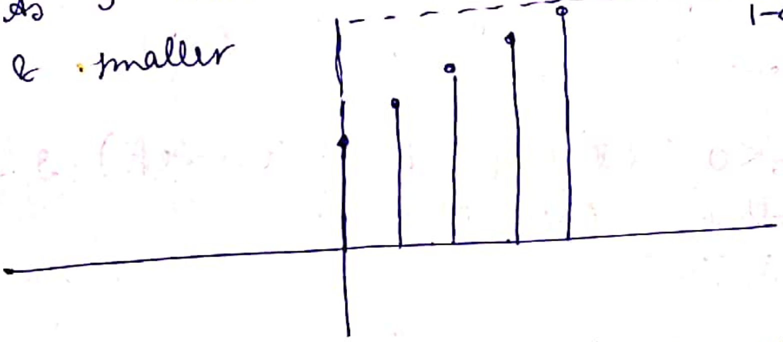
• $$\sum_{k=0}^{n} \alpha^k = \sum_{k=0}^{\infty} \alpha^k - \boxed{\sum_{k=n+1}^{k=\infty} \alpha^k} \rightarrow \alpha^{n+1} \sum_{k=0}^{\infty} \alpha^k$$

$$= \frac{1}{1-\alpha} - \frac{\alpha^{n+1}}{1-\alpha} = \boxed{\frac{1-\alpha^{n+1}}{1-\alpha}}$$

$$y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u[n]$$

As I move ahead, the O/P is getting smaller & smaller

$\frac{1}{1-\alpha}$ (never reaches this asymptote) $n \to \infty$ ($\alpha < 1$)



This should be the same as raw convolution sum

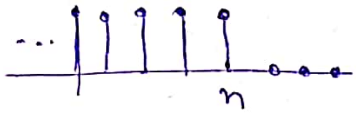
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k u[k] . u[n-k]$$

- Its good to think like what do these step funcns. & what does their product look like

- funcn of k
$u[k] = 0$ k is negative

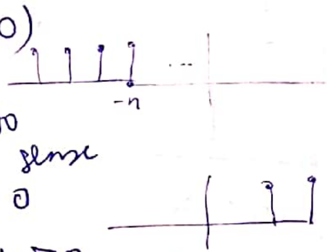
$$\sum_{k=0}^{\infty} \alpha^k u[n-k]$$



n

$$= \left(\sum_{k=0}^{n} \alpha^k\right) u[n]$$

($\because n > 0$) for the sum to make sense $k > 0$



$u[n-k] = 1$ for $n-k \geq 0$ or $\boxed{k \leq n}$

sum doesn't dip below lower index (i.e $k=0$)

<u>Properties of LTI systems</u>:-

① An LTI system is entirely determined by its impulse response.
This is not true for non-LTI systems

eg $y[n] = n\,x[n]$ (In particular not time invar.)

$\delta[n] \longrightarrow n\,\delta[n] = 0$

Impulse response is zero

For LTI then I'd never get non-zero O/P for any I/P signal but for non-LTI its not so

(power of LTI sys.)

don't have to figure what happens to other signals

(But remem. I can do this things also if I've other responses like step)

② Commutative

$$x[n] * h[n] = h[n] * x[n]$$

interchange the order of the signals

$$\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

I keep x as it is
flip h & run h across the signal

simple change of var.

$m = n - k$ If $k = -\infty$ to ∞

$k = n - m$ $m = -\infty$ to ∞

$$\sum_{m=-\infty}^{\infty} x[n-m]\,h[m]$$

③ Distributive Property

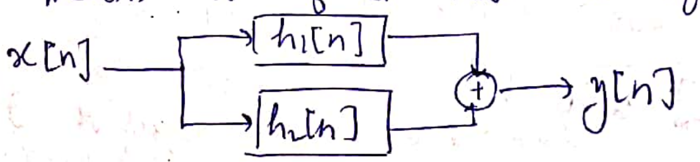
$$x[n] * (h_1[n] + h_2[n])$$

$$= x[n] * h_1[n] + x[n] * h_2[n]$$

go through 2 systems in ||

falls out from LTI prop. of sys.

what this means is if I have a signal that goes through



an eq. sys. is this

$x[n] \longrightarrow \boxed{h_1 + h_2} \longrightarrow y[n]$

or ~~If I add up 2 signals~~ combined I.R

④ Associative Property

$$x[n] * (h_1[n] * h_2[n])$$
$$= (x[n] * h_1[n]) * h_2[n]$$

$$x[n] \longrightarrow \boxed{h_1} \longrightarrow \boxed{h_2} \longrightarrow y[n]$$

$$x[n] \longrightarrow \boxed{h_1 * h_2} \longrightarrow y[n]$$

From commutative prop.

$$x[n] \longrightarrow \boxed{h_2 * h_1} \longrightarrow y[n]$$

$$x[n] \longrightarrow \boxed{h_2} \longrightarrow \boxed{h_1} \longrightarrow y[n]$$

It doesn't matter which order I put the systems in

Conclusions:-

①, Combine all my LTI syst. into 1 block

② put the sys. in any order I want and get the same answer

(not true for non-linear

$$x \longrightarrow \overset{\text{F.T}}{\boxed{2}} \longrightarrow \overset{\text{F.T}}{\boxed{x^2}} \longrightarrow (2x)^2 = 4x^2$$

$$x \longrightarrow \boxed{x^2} \longrightarrow \boxed{2} \longrightarrow 2x^2$$

not linear

⑤ Causality:-

when impulse resp. of the sys. is zero for values less than 0.

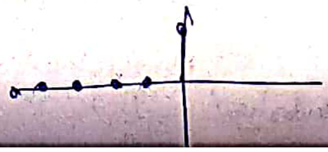
$y[n]$ doesn't depend on $x[n+k]$ for $k>0$

(not looking into future!)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

→ If k is negative I'm looking at future values

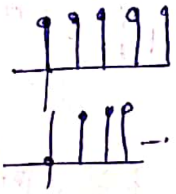
For causal system $k<0$ $h[k]$ must be zero to make those zero



$$\boxed{h[k] = 0 \qquad k<0}$$

other way of looking at it is that the impulse response has to be zero before impulse occurs → system cannot be doing stuff before it sees the impulse

6. Step Response



$$\delta[n] = u[n] - u[n-1]$$

$$\downarrow \qquad\qquad \downarrow$$

$$h[n] = s[n] - s[n-1] \quad \text{(by LTI)}$$

step resp. $\mathcal{H}(u[n])$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

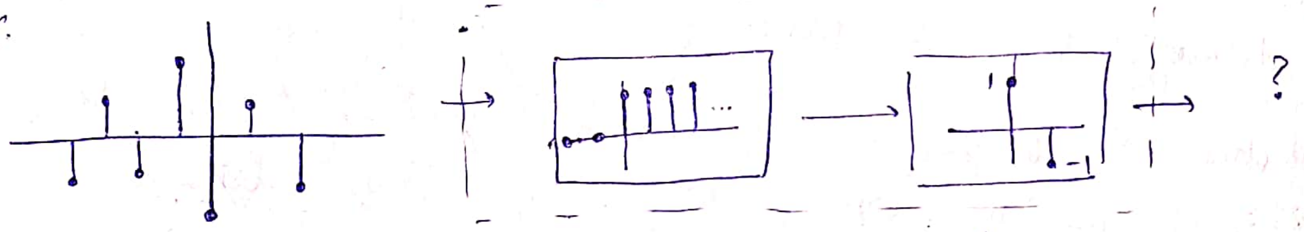
i.e if I know the step resp. I can get I.R & vice versa

→ Step Response also charac. the LTI sys.

∴ I can compute the I.R of the system if I know the step resp.

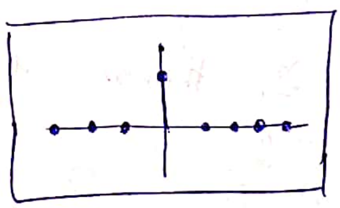
· ~~STEP response also charac~~ make life easier!

eg.



To do this one has to convolve 1st with 2nd & then the result with 3rd

But convolution of the 2 systems is easy (flip & see overlap)



(Identity func)

Basic motive is that can I simplify the system to be a bit easier

- Many electrical & mechanical systems are described by differential eqns. (Mass Spring, RLC)
- Discrete versions of these are called __Difference equations__

$$\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]$$

called Linear const. Coeff. Difference eqn

Soln will be of the form

$$y[n] = y_h[n] + y_p[n]$$

homog. soln. — particular soln.

Simpler case $y[n] = \sum b_k \, x[n-k]$ → F.I.R Finite Imp. Sys.

there is nothing to solve here, I use convolution to reach to O/P (seen doing these type of problems)

- But if I've previous values of y, then it'll be a sort of recursive eqn. & then go thru machinery of homog. soln. & particular.

- convolution & Responses of LTI systems can be greatly simplified with tools of Freq. domain analysis (Fourier & Z-transform)
- In C.T, Laplace Transform was the best way to solve differential eqns.
- Same here once Z trans. life is easy.

__Lec 4:-__

__Fourier Series:-__

(• Fourier Analysis → decomposition of signals into sines & cosines (why sines & cosines? It turns out that they naturally occur in a lot of our systems)
- mechical systems → swinging pend. & spinning wheels
- communication → cellphones, radio etc built on carrier waves (high freq. sin)

- Power signals

(• nice thing is that LTI systems respond in a particular way to sinusoidal I/P)

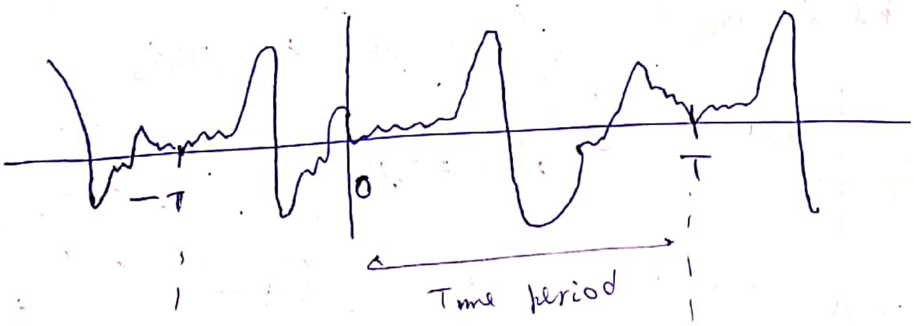
Principle.

- Every <u>periodic</u> C.T signal can be written as a sum of <u>sinusoids</u>.

(Aperiodic → $T \to \infty$)

- Assume we have a periodic signal $x(t)$

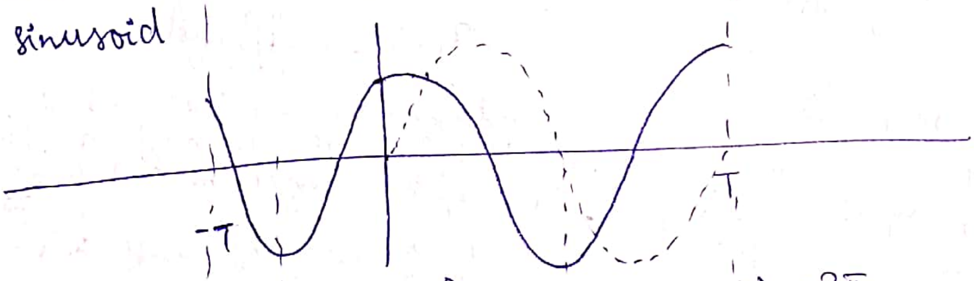
$$x(t+T) = x(t)$$



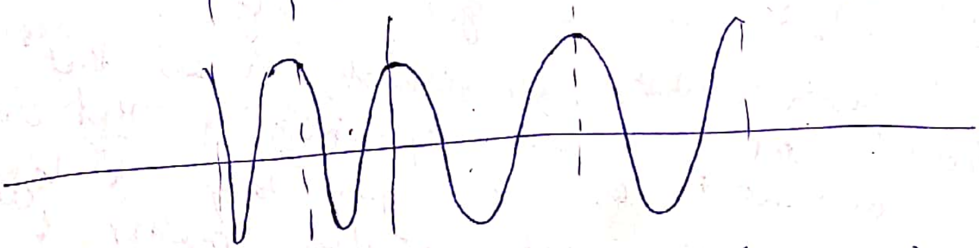
1st Question is?

What other signals have the same period & T?

a most natural signal that has a period is a sinusoid



$\cos(\omega_0 T)$ $\quad$ $\omega_0 = \frac{2\pi}{T}$ → Fundamental freq.



$\cos(2\omega_0 T)$ or $\cos\left(2 \cdot \frac{2\pi}{T} \cdot t\right)$

Thats not the only freq. that has period T. I can also think about freq. that wiggle faster

$\cos k\omega_0 t$, k an integer has this period

Also, $\sin k\omega_0 t$,

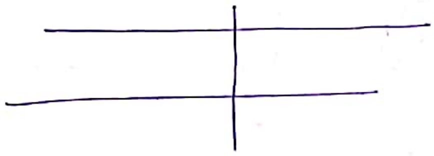
Also, $e^{jk\omega_0 t} = \cos k\omega_0 t + j \sin k\omega_0 t$ is also periodic (Real & Img.) with period T

$\sim$ ly. $\displaystyle\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ combination of such signals with period T is also periodic with period T.

↑ Coeff. (Any complex #)
(because it won't change its period)
(only amplitude & phase)

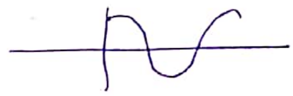
Philosophy of what we're gonna do...
We're gonna take our signal $x(t)$ & make it look like a bunch of such sinusoids with increasing freq...
(How to know these a_k)

for $k=0$

a_0 ·  "DC"

$+ \quad a_1 \cdot (\cos \omega_0 t + j \sin \omega_0 t)$

$+ \quad a_{-1} \, (\cos(-\omega_0 t) + j \sin(-\omega_0 t))$
↳ $a_{-1} \, (\cos \omega_0 t - j \sin \omega_0 t)$

$a_{\pm 1}$ · 

$a_{\pm 2}$ ·  higher freq.

$a_{\pm 3}$ · 

→ our goal is to represent

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}} \quad (A) \longleftarrow \boxed{\text{Fourier Series}}$$

'synthesis'

Interpretation is that as I ↑ k I get the coeff. of things that wiggle more & more (synthesise $x(t)$)?

How to compute $\{a_k\}$ for a given $x(t)$?

* It stands for reason that for most signals that we care about the amount of wiggling that I've to take into account as k gets bigger & bigger at some pts these coeff. get probably smaller & smaller because a real world signal prob. doesn't wiggle like crazy fast.
We just need to have certain ~~sound~~ sums of coming that we need to add up.

↳

We're gonna assume initially that this property holds

multiply b/s by $e^{-jn\omega_0 t}$

$$x(t).\, e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} . e^{-jn\omega_0 t}$$ (n is a fixed integer)

$$= \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}$$ (collect these 2 terms together)

Integrate b/s from 0 to T

$$\int_0^T x(t) e^{-jn\omega_0 t} dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \underline{\int_0^T e^{j(k-n)\omega_0 t} dt}$$ (switch the order of sum & integral) well behaved

now,

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \int_0^T \cos (k-n)\omega_0 t \, dt + j \int_0^T \sin (k-n)\omega_0 t \, dt$$ Diff. to visualize comp. integral just to make a bit easier

$$\int_0^T 0 \, dt$$

now, k and n are integers

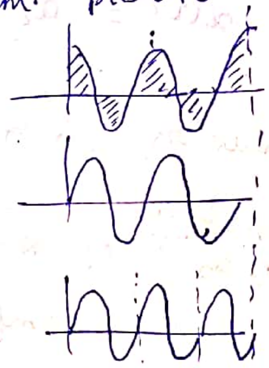
• say $k=n$ $= \int_0^T 1 \, dt = T$

∴ For $k=n$, The integral is T

• say $k \neq n$

$$\int_0^T \cos(\text{integer})\omega_0 t \, dt + j \sin(\text{integer}) T$$ Rem. picture

oscillates an eq. no. of times inside $[0,T]$ ⇒ integral is 0



Thus, $\int_0^T x(t) e^{-jn\omega_0 t} = a_n T$

∴ a_n ~~= a~~ $\boxed{a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt}$ Analysis eqn. How do I get a sp. coeff.

$a_0 = \frac{1}{T}\int_0^T x(t)\, dt$ → DC or Avg. $\int_{-T/2}^{T/2}$

$\{a_k\}$ are the Fourier series coeffient of $x(t)$ (spectral coeff.)

(In gen. $x(t)$ can be a complex valued signal) & a_k are complex

→ what if $x(t)$ is real ? $\{a_k\}$ are ~~res~~ complex, but there're patterns

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

complex conjugate,

$$x^*(t) = \sum_{k=-\infty}^{\infty} a_k^* e^{-jk\omega_0 t}$$

If z is real $z = z^*$

$$= \sum_{l=-\infty}^{\infty} a_{-l}^* e^{jl\omega_0 t}$$

Reorder numbers

chang there ~~the~~ ~~use~~ of variables for comparison

$$= \sum_{k=-\infty}^{\infty} a_{-k}^* \cdot e^{jk\omega_0 t}$$

$\boxed{(a_k = a_{-k}^*)}$ $\because x(t) = x^*(t)$

So, $a_1 = 1+2j$

$a_{-1} = 1-2j$

∴ keep track of +ve a_k & I can immed- check what -ve a_k are

For $x(t)$ real, we can also write

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2 \underset{\text{amplitude}}{\overset{\text{real}}{A_k}} \cos(\underset{\text{phase}}{\theta_k} + k\omega_0 t)$$

(• the intuition is that I should be able to make that signal out of a bunch of real cosines or sinusoids) eg. to get a sine way we need to shift the sinusoids to get 0 at $t=0$.

These are computable from the $\{a_k\}$

Alternatively (for no phase shift)

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} \overset{\text{real}}{B_k} \cos k\omega_0 t - \overset{\text{real}}{C_k} \sin k\omega_0 t$$

(related to a_k)

, a bunch of non shifted sines & cosines (only for real eg:

Fast Fourier xform (complex exponentials)

$$x(t) = 5 + 2 \cos(\omega_0 t)$$

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

• note when u've something i.e strictly visibly a sum of cosines, one can immediately figure out what the Fourier series is since

$$x(t) = 5 + 2\left(\frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right)$$

$$= 5 + e^{j\omega_0 t} + e^{-j\omega_0 t}$$

$$\downarrow$$

$$a_0 e^{0.j\omega_0 t} + a_1 e^{1.j\omega_0 t} + a_{-1} e^{-1j\omega_0 t} + \ldots$$

$\therefore$

$a_0 = 5$

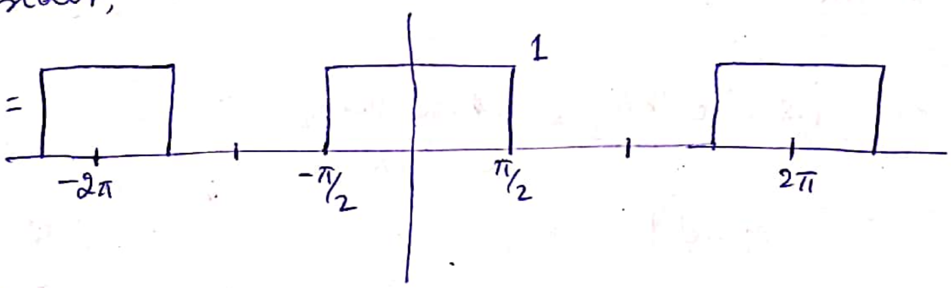
$a_1 = 1$

$a_{-1} = -1$ (signal is real ; obey complex conjugate prop.)

All other $a_k = 0$

Lets consider,

$x(t) =$



Fundamental

$T = 2\pi$, $\omega_0 = \frac{2\pi}{T} = 1$

DC term

$$a_0 = \frac{1}{T}\int_0^T x(t).\, \underline{e^{-j0.\omega_0 t}}\, dt = \frac{1}{T}\int_0^T x(t)\, dt$$

= Average value of the signal over one period

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\, dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1.\, dt = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x(t)\, dt$$

(same stuff inside) simpler single piece

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\, e^{-jkt}\, dt$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{-jkt}\, dt$$

$$= \frac{1}{2\pi} \cdot \frac{-1}{jk} \cdot e^{-jkt}\Big]_{-\pi/2}^{\pi/2}$$

$$= \frac{-1}{2\pi jk}\left(e^{-jk\pi/2} - e^{jk\pi/2}\right)$$

$$= \frac{1}{\pi k}\left(\frac{e^{jk\pi/2} - e^{-jk\pi/2}}{2j}\right) = \frac{\sin k\pi/2}{\pi k}$$

$$a_1 = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} \quad ; \quad a_{-1} = a_1^* = \frac{1}{\pi} \; , \quad a_2 = \frac{\sin \pi}{2\pi} = 0$$

sinc funcn. $= \frac{\sin x}{x}$
sinc x

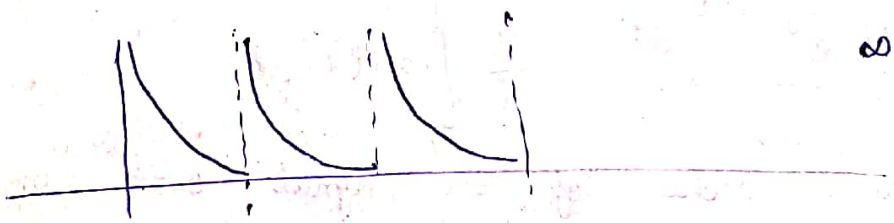
$$\frac{1}{2} \text{sinc}\, k\pi/2 = \frac{1}{2}\frac{\sin k\pi/2}{k\pi}$$

$\boxed{\text{Fourier Series Applet}}$ → falstad.com

→ notes & Properties of the Fourier Series :-

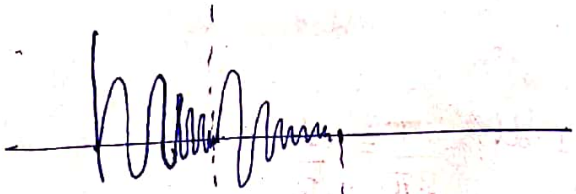
- when doesn't it work?

(1)



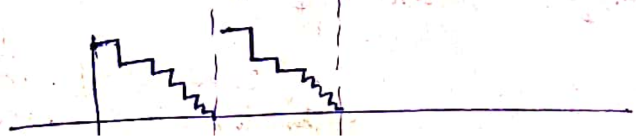
∞. area under the curve

(2)



∞ - & wiggling

(3).

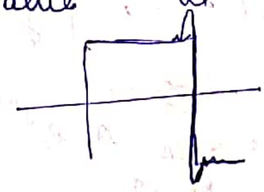


∞ discontinuity

What happens at a discontinuity?

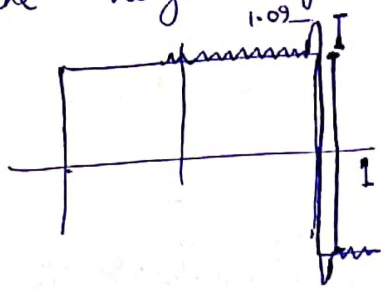
(a) F.S converges at every continous pt.

(b) F.S converges to the avg. value at every discontinuity (say square wave



GIBBS phenomenon

never get the overshoot below ≈ 9% of the height of discontinuity



Properties:-

We've $x(t)$ periodic with period T

F.S $\{a_k\}$ $\qquad x(t) \longleftrightarrow a_k$

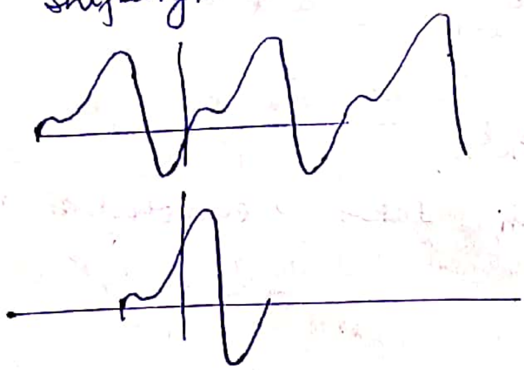
① Linearity

$x(t) \longleftrightarrow \{a_k\}$ $\qquad$ have same period T

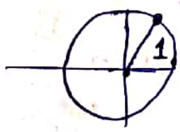
$y(t) \longleftrightarrow \{b_k\}$

$$\alpha\, x(t) + \beta\, y(t) \longleftrightarrow \{\alpha a_k + \beta b_k\}$$

② Time Shifting:-



$x(t) \longleftrightarrow \{a_k\}$



mag. is always 1 (or same)

let $y(t) = x(t - t_0)$

$y(t) \longleftrightarrow a_k e^{-jk\omega_0 t_0}$

ie a phase shift of F.S coeff. & the magnitude doesn't change

$|b_k| = |a_k|$

③ Differentiation

$$x(t) \longleftrightarrow \{a_k\}$$

$$x'(t) \longleftrightarrow \{jk\omega_0 a_k\}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$x'(t) = \sum_{k=-\infty}^{\infty} (a_k \cdot jk\omega_0) e^{jk\omega_0 t}$$

Triangular wave $\xrightarrow{\text{diff}}$ square wave

④ Parseval's theorem

$$\frac{1}{T}\int_0^T |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

Power signal → • Energy ∞ • exist over ∞ time • periodic

energy signal → power is 0 • time limited • non-periodic

avg. power of the signal = power of F.S coeff

(same amount of power in time as in freq. domain)

or no inform. lost by converting

⑤ Convolution

$$x(t) \longleftrightarrow \{a_k\}$$

$$y(t) \longleftrightarrow \{b_k\}$$

$$x(t) \cdot y(t) \longleftrightarrow \sum_{l=-\infty}^{\infty} a_l \cdot b_{k-l} = a * b$$

or

$$\int_0^T x(\tau) \cdot y(t-\tau) \, d\tau \longleftrightarrow T a_k b_k$$

multiplication in one domain takes convolution in another domain

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Tutorial I

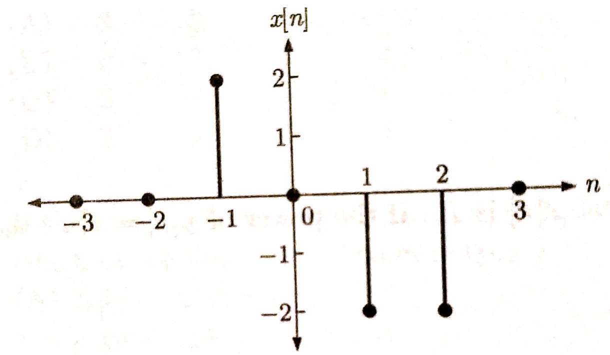
Course Title: Digital Signal processing — Semester: Sixth (6^{th})
Date: 23.04.2020
Course Code: ELE-605

I. Discrete-Time Fourier Transform

Q.1) Find the even and odd parts of the following signals:

1. x[n]= (6, **4**↑, 2, 2)
2. x[n]= (-4, 5, 1, **-2**↑, -3, 0, 2)
3. x[n]= $a^{|n|}$
4. x[n]= $na^n u[n]$

Q.2) Consider a signal x[n] as shown in the figure below



1. If x[n] is transformed into $y[n] = \frac{2}{3}x[-n-2] - 2$, y[n] is
2. What is y[n]=x[-n/3]

Q.3) Determine whether or not each of the following sequences is periodic. If your answer is yes, determine the period.

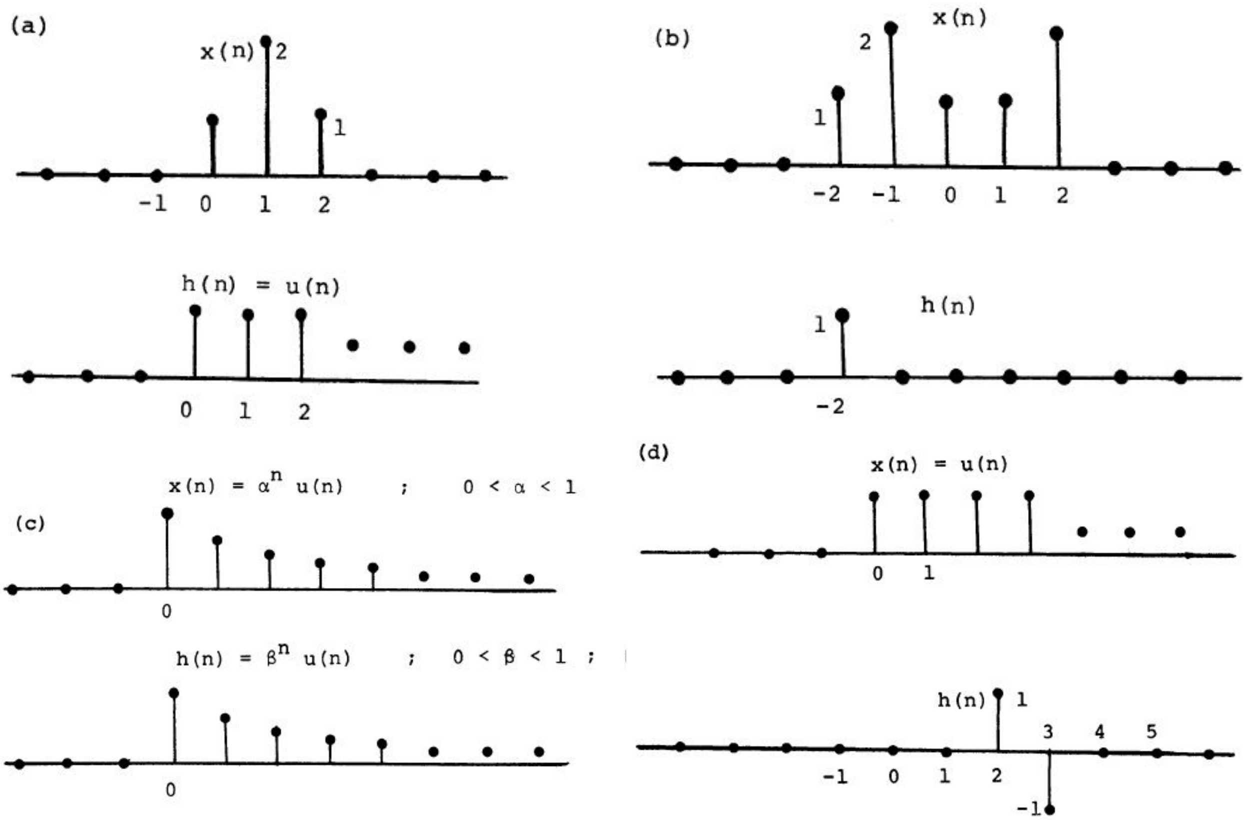
1. x[n]= $A\cos\left(\frac{3\pi}{7}n - \frac{\pi}{8}\right)$
2. x[n]= $e^{j\left(\frac{n}{8} - \pi\right)}$

Q.4) For each of the following systems, y(n) denotes the output and x(n) the input. Determine for each whether the specified input-output relationship is linear, shift-invariant and causal.

1. y[n]= $2x[n] + 3$
2. y[n]= $x[n]\sin\left(\frac{2\pi}{7}n + 6\right)$

3. y[n]=$(x[n])^2$
4. y[n]= $\sum_{m=-\infty}^{n} x[m]$

Q.5) For each of the following pairs of sequences, x(n) represents the input to an LTI system with unit-sample response h(n). Determine each output y(n). Sketch your results.



Q.6) The system shown below contains two LTI subsystems with unit sample responses h_1 (n) and h_2 (n), in cascade. Consider x[n] as a unit step.