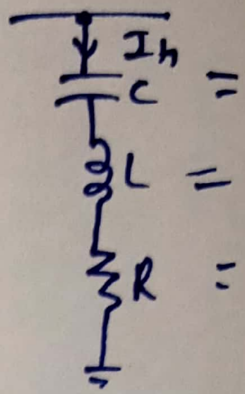
Design of Harmonic Filters:

Single-Tuned Filter:



Filter impedance is,

$$Z_f = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Now, $\omega_n = \frac{1}{\sqrt{LC}}$

Frequency deviation,

$$\delta = \frac{\omega - \omega_n}{\omega_n}$$

$$\delta\omega_n = \omega - \omega_n$$

$$\boxed{\omega = \omega_n(1+\delta)}$$

Reactance at resonant frequency,

$$X = \omega_n L = \sqrt{\frac{L}{C}}$$

$$\therefore Z_f = R + j\left(\frac{\omega L}{\omega_n} - \frac{1}{\omega_n \omega C}\right) \cdot \omega_n$$

$$Z_f = R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega C R}\right)\right]$$

Now, $Q = \frac{\omega_n L}{R} = \frac{1}{\omega_n C R}$

Quality factor

In terms of quality factor,

$$Z_f = R\left[1 + jQ\left(\frac{\omega}{\omega_n} - \frac{\omega_n}{\omega}\right)\right]$$

As $\omega = \omega_n(1+\delta)$

$$\therefore Z_f = R\left[1 + jQ(1+\delta) - \frac{1}{(1+\delta)}\right]$$

$$\boxed{Z_f = R\left[1 + jQ\delta\left(\frac{2+\delta}{1+\delta}\right)\right]}$$

As δ is very smaller,

$$\therefore \boxed{Z_f = R[1 + j2Q\delta]}$$

Also, $\boxed{Z_f = X_0 K}$

Where $X_0 = \sqrt{\frac{L}{C}}$ &

$$K = \frac{1}{Q} + j\delta\left(\frac{2+\delta}{1+\delta}\right)$$

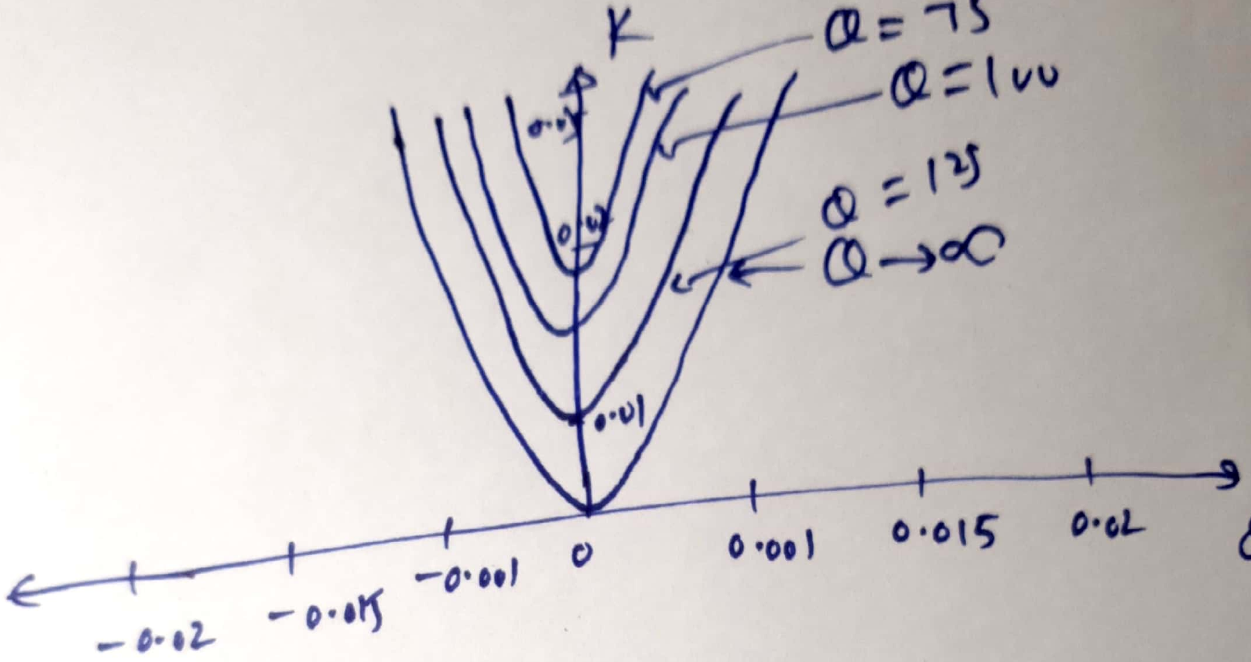
For $Q \to \infty$,

$$\boxed{K = j\delta\left(\frac{2+\delta}{1+\delta}\right)}$$

$$\boxed{Z_f = K X_0}$$

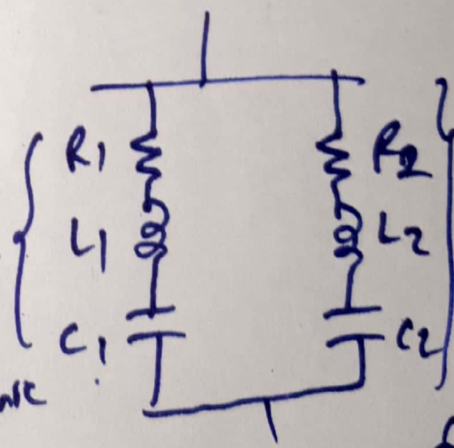
$$X_0 = \sqrt{\frac{L}{C}}$$

$$K = j\delta\left(\frac{2+\delta}{1+\delta}\right) + \frac{1}{Q}$$



Double-Tuned Filter:

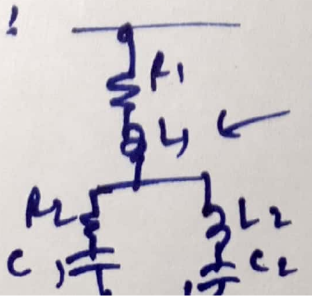
i) Composed of Two Single-Tuned Filters:



Tuned to one harmonic freq.

Tuned to Two harmonic freq.

ii) Single Double-Tuned filter:



Double-tuned filter

Now,

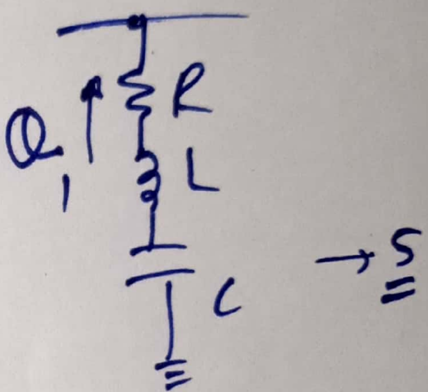
$$\delta = \frac{\omega - \omega_n}{\omega_n}$$

$$\delta = \frac{\omega}{\omega_n} - 1 \qquad \omega_n = \sqrt{\frac{L}{C}}$$

$$\delta = \omega\sqrt{\frac{C}{L}} - 1$$

$$\Delta\delta = \frac{\Delta\omega}{\omega} + \frac{1}{2}\left(\frac{\Delta C}{C} + \frac{\Delta L}{L}\right)$$

Reactive Power compensation:



Reactive power,

$$Q = I_1^2 (X_L - X_C)$$

$$Q = I_1^2 \left(\omega_1 L - \frac{1}{\omega_1 C}\right)$$

$$Q = I_1^2 \omega_1 L\left(1 - \frac{1}{\omega_1^2 LC}\right) \quad -(1)$$

As $\omega_n = \frac{1}{\sqrt{LC}}$ $\quad\therefore \omega_n^2 = \frac{1}{LC}$

$$\therefore Q = I_1^2 \omega_1 L\left(1 - \frac{\omega_n^2}{\omega_1^2}\right)$$

$$Q = I_1^2 \omega_1 L\left(1 - \frac{\omega_n^2}{\omega_1^2}\right) \quad \text{--- (2)}$$

$$\omega_n = h\omega_1$$

∴ From eqn. (2),

$$Q = I_1^2 \omega_1 L\left(1 - \frac{h^2 \cancel{\omega_1^2}}{\cancel{\omega_1^2}}\right)$$

$$\Rightarrow \boxed{Q = I_1^2 \omega_1 L\,(1 - h^2)}$$

For a 6-pulse converter,

$h = 5, 7, 11, 13, 17, 19, 23, 25 \ldots$

For a 12-pulse converter,

$h = 11, 13, 23, 25, \ldots$

$(1-h^2) \rightarrow -$ive

$\Rightarrow Q \rightarrow -$ive

⇒ Reactive power is generated by harmonic filter & injected into the system for some reactive-power compensation.

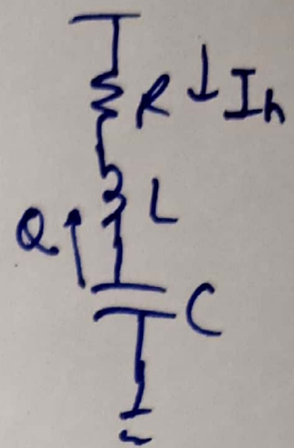
∴ Cost of Harmonic Filter is 5% to 10% of cost of terminal equipment.

Scanned with CamScanner

Minimum Cost Tuned – Filter Design:

Rating of tuned capacitor is given by reactive power (MVARs) supplied by it.

∴ Reactive power generated by capacitor,



$$Q_{cr} = V_1^2 \omega C + I_h^2 X_{ch}$$

$$Q_{cr} = V_1^2 \omega_1 C + \frac{I_h^2}{h\omega_1 C}$$

Let $S = V_1^2 \omega_1 C = \frac{I_1^2}{\omega_1 C}$

$$\therefore Q_{cr} = S + \frac{V_1^2 I_h^2}{V_1^2 \omega_1 C h}$$

$$\therefore \boxed{Q_{cr} = S + \frac{V_1^2 I_h^2}{S_h}} \quad -(1)$$

$\rightarrow$ MVARs

$$V_1^2 \omega_1 C h = S_h$$

$$Q_c = \frac{V^2}{X_c} = \frac{V^2}{1/\omega C} = V^2 \omega C$$

$$X_c = \frac{1}{\omega C}$$

$$X_{ch} = \frac{1}{h\omega_1 C}$$

This determines the MVAR rating of filter capacitor..

Rating of Tuned Inductor:

Reactive power,

$$Q_{Lr} = I_1^2 \omega_1 L + I_h^2 X_{Lh}$$

$$\rightarrow I_h^2 X_{Lh}$$

$$Q_{Lr} = I_1^2 \omega_1 L + I_h^2 h \omega_1 L \quad -(2)$$

Now, $\omega_n = \dfrac{1}{\sqrt{LC}}$ $\quad (\because X_{Lh} = h\omega_1 L)$

$$\omega_n^2 = \frac{1}{LC}$$

$$L = \frac{1}{\omega_n^2 C} = \frac{1}{h^2 \omega_1^2 C}$$

$\therefore$ From eqn. (2), we have

$$Q_{Lr} = I_1^2 \omega_1 \cdot \frac{1}{h^2 \omega_1^2 C} + I_h^2 h \omega_1 \cdot \frac{1}{h^2 \omega_1^2 C}$$

$$Q_{Lr} = \frac{I_1^2}{h^2 \omega_1 C} + \frac{I_h^2}{h \omega_1 C}$$

$$Q_{Lr} = \frac{S}{h^2} + \frac{V_1^2 I_h^2}{h V_1^2 \omega_1 C}$$

$$\boxed{Q_{Lr} = S/h^2 + \frac{V_1^2 I_h^2}{hS}} \rightarrow \text{MVARs} \quad -(3)$$

$\therefore$ Total cost of Tuned Filter,

$$\boxed{K = Q_{rc} U_c + Q_{rL} U_L} \quad — (4)$$

where U_c & U_L are per unit cost (dollars/MVAR) of filter capacitor and inductor respectively.

$$K = \left(S + \frac{V_1^2 I_h^2}{S h} \right) U_c + \left(\frac{S}{h^2} + \frac{V_1^2 I_h^2}{S h} \right) U_L$$

$$K = S U_c + \frac{V_1^2 I_h^2}{S h} \cdot U_c + \frac{S}{h^2} U_L + \frac{V_1^2 I_h^2}{S h} \cdot U_L$$

$$\boxed{K = S\left(U_c + \frac{U_L}{h^2} \right) + \frac{V_1^2 I_h^2}{h S}\left(U_c + U_L \right)}$$

$$\Rightarrow K = A S + \frac{B}{S}$$

For minimum cost, $\dfrac{dK}{dS} = 0$

$$\Rightarrow \frac{d}{dS}\left(A S + \frac{B}{S} \right) = 0$$

$$A - \frac{B}{S^2} = 0$$

$$\Rightarrow \boxed{S = \sqrt{\frac{B}{A}}}$$

$$S = \sqrt{\frac{B}{A}} \rightarrow \text{condition for min. cost of filter.}$$

$\therefore$ Minimum cost,

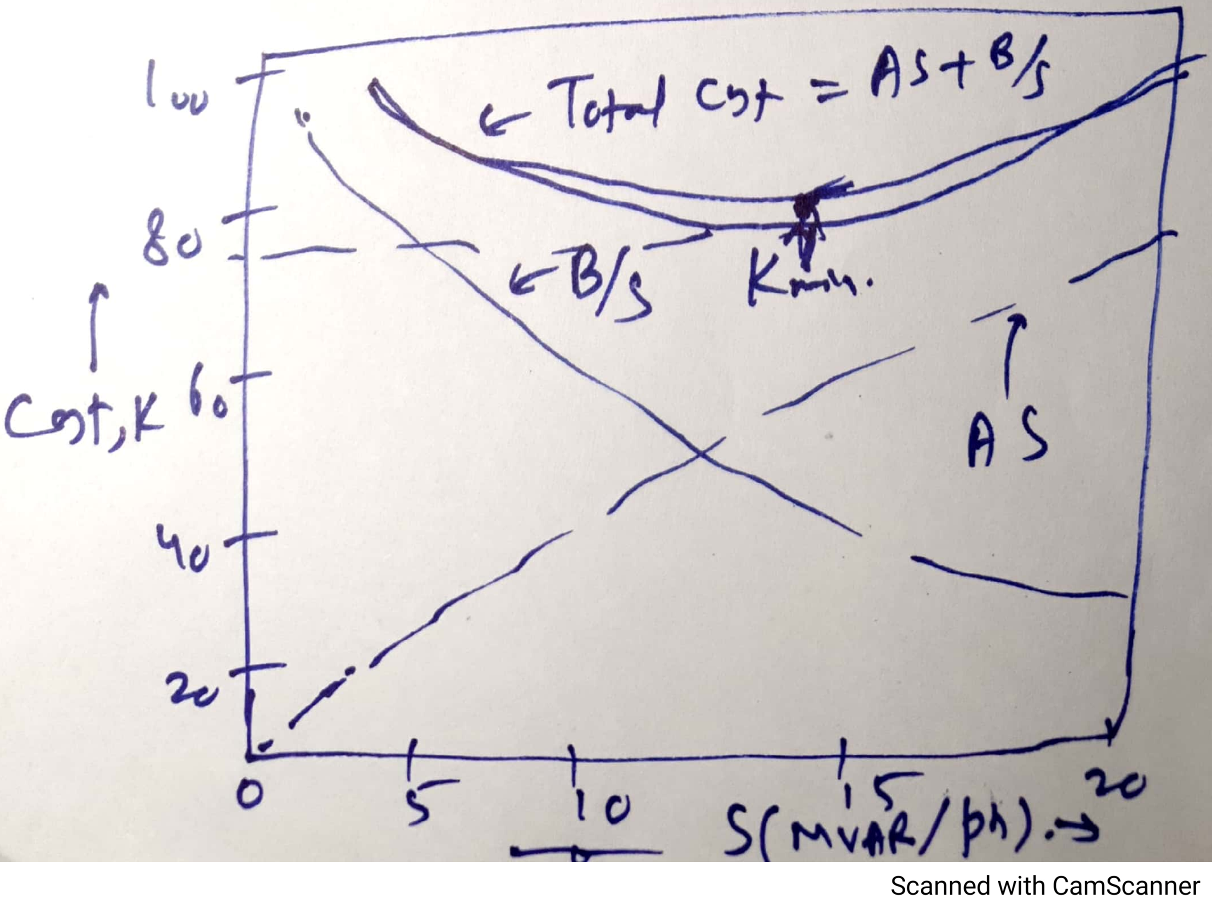
$$K_{min.} = AS + \frac{B}{S}$$

$$K_{min.} = A \cdot \frac{\sqrt{B}}{\sqrt{A}} + B \times \frac{\sqrt{A}}{\sqrt{B}}$$

$$K_{min} = A \frac{\sqrt{B}}{\sqrt{A}} \times \frac{\sqrt{A}}{\sqrt{A}} + B \times \frac{\sqrt{A}}{\sqrt{B}} \times \frac{\sqrt{B}}{\sqrt{B}}$$

$$K_{min.} = \sqrt{AB} + \sqrt{AB}$$

$$\Rightarrow \boxed{K_{min.} = 2\sqrt{AB}}$$



Ex.

Let $V_d = \pm 300V$

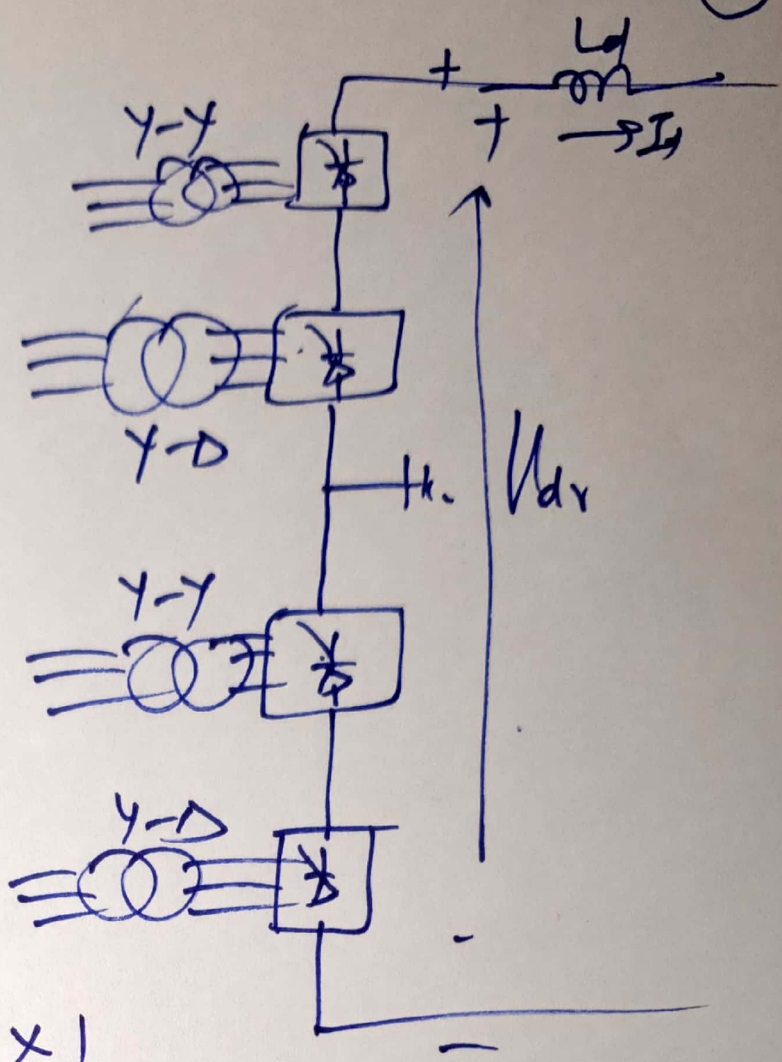
(Bipolar Link)

4 Bridges/pole

12-pulse operation.

Let $V_L = 235 kV$

$I_d = 1 kA$



Power Transmitted,

$P_d = V_d \times I_d = 600 \times 1$

$P_d = 600 MW$ Let pf = 0.866 (given)

$\therefore P_{ac} = 3 \times V_1 \times I_{L1} \cos\phi$

$600\,MW = 4 \times 3 \times \frac{235}{\sqrt{3}} \times I_{L1} \times 0.866$

$\Rightarrow I_{L1} = \frac{1700}{4} A$ (rms)

Let filter be designed for suppressing 5^{th} harmonic comp.

$\therefore I_{hf} = I_5 = \frac{I_{L1}}{5} = 85A$

$U_L = ?$ $\quad U_C = ?$

$V_1 = \frac{235}{\sqrt{3}}\ KV$

let $U_c = \$3.5/KVAR$ } given
$U_L = \$8.0/KVAR$

$A = U_c + \frac{U_L}{h^2} = \left(3.5 + \frac{8}{5^2}\right)$

$A = \$3.82/KVAR$
$= \$3820/MVAR$

$B = \frac{V_1^2 I_{hf}^2}{h}(U_c + U_L)$

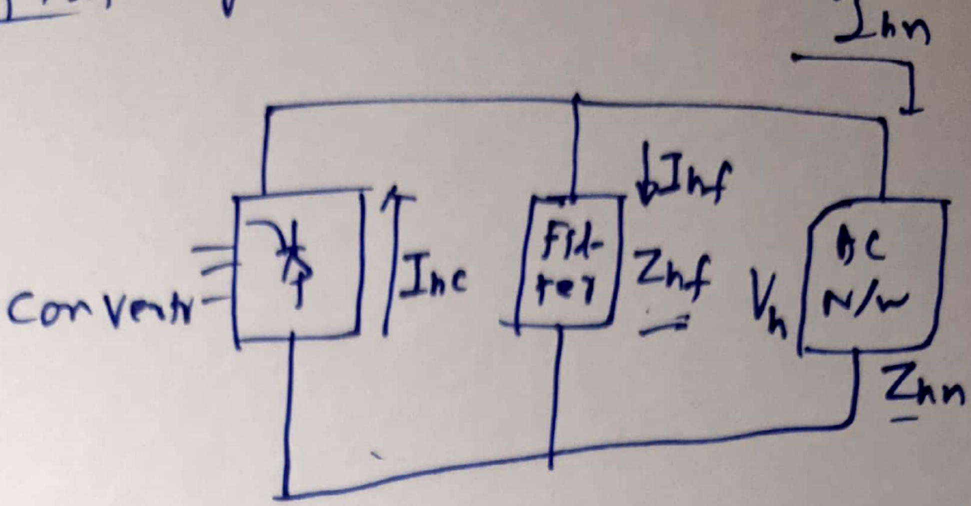
$B = \frac{(235/\sqrt{3})^2 \times (85\times10^{-3})^2}{5} \times (3500 + 8000)$

$B = \quad ?$

$\therefore S = \sqrt{\frac{B}{A}}$ $\quad \therefore K_{min.} = 2\sqrt{AB}$

$\boxed{K_{min.} = \$71,000/ph.}$

Effect of Network Impedance on Filtering:



$$V_h = I_{hn} \times Z_{hn}$$

$$V_h = \left(\frac{Z_{hf}}{Z_{hf}+Z_{hn}} \times I_{hc}\right) \times Z_{hn}$$

(By current-divider formula)

$$\boxed{V_h = \frac{I_{hc} \times Z_{hf}}{Z_{hn}+Z_{hf}} \cdot Z_{hn}}$$

- When $Z_{hn} = 0$ (Unrealistic)
  $\Rightarrow$ whole harmonic current goes to AC n/w
- When Z_{hf} = infinite ← Ideal case
  $\Rightarrow$ whole harmonic current is trapped by filter & no harmonic flows into AC n/w (Desirable)

∴ Desirable:

$Z_{nn} \gg Z_{nf}$

Z_{nn} $\downarrow$ Not in our control

Z_{nf} $\downarrow$ Controllable