Module
4

Single-phase AC Circuits

Lesson
12

Generation of Sinusoidal Voltage Waveform (AC) and Some Fundamental Concepts

In this lesson, firstly, how a sinusoidal waveform (ac) is generated, is described, and then the terms, such as average and effective (rms) values, related to periodic voltage or current waveforms, are explained. Lastly, some examples to find average and root mean square (rms) values of some periodic waveforms are presented.

Keywords: Sinusoidal waveforms, Generation, Average and RMS values of Waveforms.

After going through this lesson, the students will be able to answer the following questions:

1. What is an ac voltage waveform?
2. How a sinusoidal voltage waveform is generated, with some detail?
3. For periodic voltage or current waveforms, to compute or obtain the average and rms values, and also the time period.
4. To compare the different periodic waveforms, using above values.

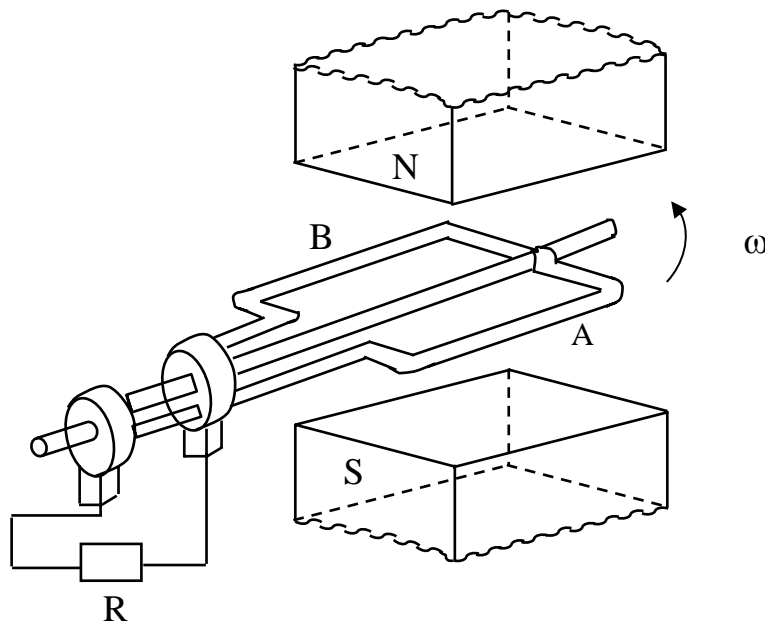# Generation of Sinusoidal (AC) Voltage Waveform

Fig. 12.1 Schematic diagram for single phase ac generation

A multi-turn coil is placed inside a magnet with an air gap as shown in Fig. 12.1. The flux lines are from North Pole to South Pole. The coil is rotated at an angular speed, $\omega = 2\pi n$ (rad/s).

$n = \dfrac{\omega}{2\pi}$ = speed of the coil (rev/sec, or rps)

$N = 60 \cdot n$ = speed of the coil (rev/min, or rpm)
l = length of the coil (m)
b = width (diameter) of the coil (m)
T = No. of turns in the coil

B = flux density in the air gap (Wb/m^2)

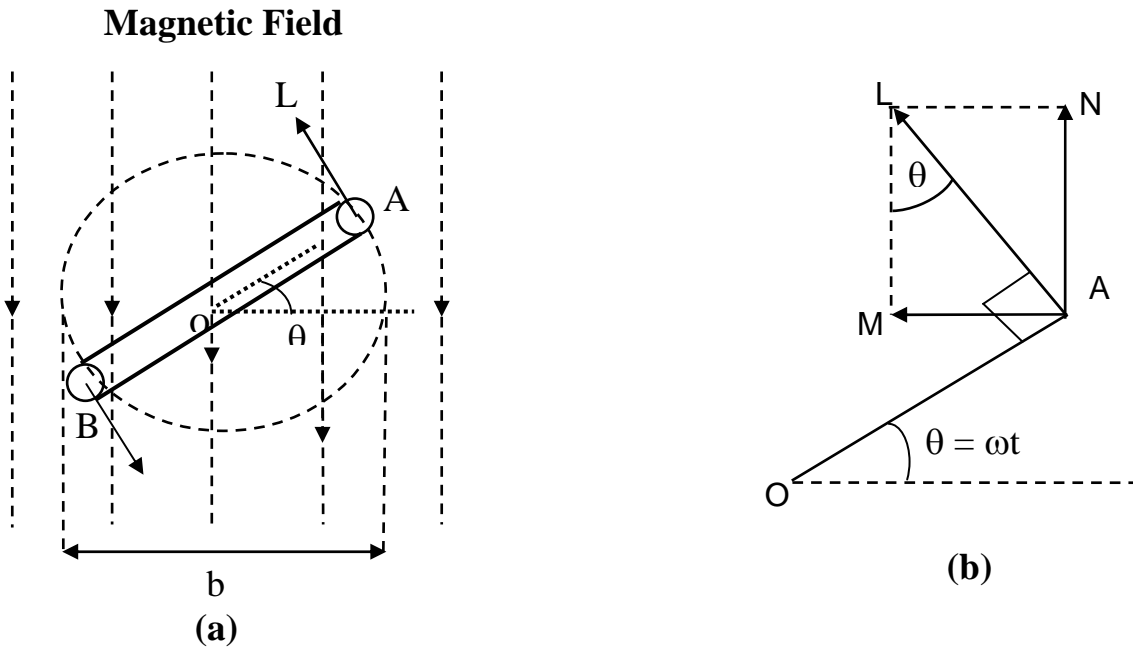$v = \pi\,b\,n$ = tangential velocity of the coil (m/sec)

Magnetic Field

Fig. 12.2 (a) Coil position for Fig. 12.1, and (b) Details

At a certain instant t, the coil is an angle (rad), $\theta = \omega t$ with the horizontal (Fig. 12.2). The emf (V) induced on one side of the coil (conductor) is $B\,l\,v\sin\theta$,

θ can also be termed as angular displacement.

The emf induced in the coil (single turn) is $2\,B\,l\,v\sin\theta = 2\,B\,l\,\pi\,b\,n\sin\theta$

The total emf induced or generated in the multi-turn coil is

$e(\theta) = T\,2\,B\,l\,\pi\,b\,n\sin\theta = 2\,\pi\,B\,l\,b\,n\,T\sin\theta = E_m\sin\theta$

This emf as a function of time, can be expressed as, $e(t) = E_m\sin\omega t$. The graph of $e(t)$ or $e(\theta)$, which is a sinusoidal waveform, is shown in Fig. 12.4a

Area of the coil $(m^2) = a = l\,b$

Flux cut by the coil (Wb) $= \phi = a\,B = l\,b\,B$

Flux linkage (Wb) $= \psi = T\,\phi = T\,B\,l\,b$

It may be noted these values of flux ϕ and flux linkage ψ, are maximum, with the coil being at horizontal position, $\theta = 0$. These values change, as the coil moves from the horizontal position (Fig. 12.2). So, also is the value of induced emf as stated earlier.

The maximum value of the induced emf is,

$$E_m = 2\,\pi\,n\,B\,l\,b\,T = 2\,\pi\,n\,\phi\,T = 2\,\pi\,n\,\psi = \omega\,\psi = \psi\,\frac{d\theta}{dt}$$

Determination of frequency (f) in the ac generator

In the above case, the frequency (Hz) of the emf generated is

$f = \omega /(2\pi) = n$, no. of poles being 2, i.e. having only <u>one</u> pole pair.

In the ac generator, no. of poles = p, and the speed (rps) = n, then the frequency in Hz or cycles/sec, is

f = no. of cycles/sec = no. of cycles per rev × no. of rev per sec = no. of pairs of poles × no. of rev per sec = $(p/2)\,n$

or, $f = \dfrac{p\,N}{120} = \dfrac{p}{2} \cdot \dfrac{\omega}{2\pi}$

Example

For a 4-pole ac generator to obtain a voltage having a frequency of 50 Hz,

the speed is, $n = \dfrac{2\,f}{p} = \dfrac{2 \times 50}{4} = 25\ \text{rps} = 25 \cdot 60 = 1,500\ \text{rpm}$

For a 2-pole (p = 2) machine, the speed should be 3,000 rpm.

Similarly, the speed of the machine having different no. of poles, required to generate a frequency of 50 Hz can be computed.

Sinusoidal voltage waveform having frequency, f with time period (sec), $T = 1/f$

Periodic Voltage or Current Waveform

Average value

The current waveform shown in Fig. 12.3a, is periodic in nature, with time period, T. It is positive for first half cycle, while it is negative for second half cycle.

The average value of the waveform, $i(t)$ is defined as

$$I_{av} = \frac{Area\ over\ half\ cycle}{Time\ period\ of\ half\ cycle} = \frac{1}{T/2} \int_{0}^{T/2} i(t)\ dt = \frac{2}{T} \int_{0}^{T/2} i(t)\ dt$$

Please note that, in this case, only half cycle, or half of the time period, is to be used for computing the average value, as the average value of the waveform over full cycle is zero (0).

If the half time period (T/2) is divided into 6 equal time intervals (ΔT),

$$I_{av} = \frac{(i_1 + i_2 + i_3 + \cdots i_6)\,\Delta T}{6 \cdot \Delta T} = \frac{(i_1 + i_2 + i_3 + \cdots i_6)}{6} = \frac{Area\ over\ half\ cycle}{Time\ period\ of\ half\ cycle}$$

Please note that no. of time intervals is n = 6.

Root Mean Square (RMS) value

For this current in half time period subdivided into 6 time intervals as given above, in the resistance R, the average value of energy dissipated is given by

$$\propto \left[\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_6^2)}{6} \right] R$$
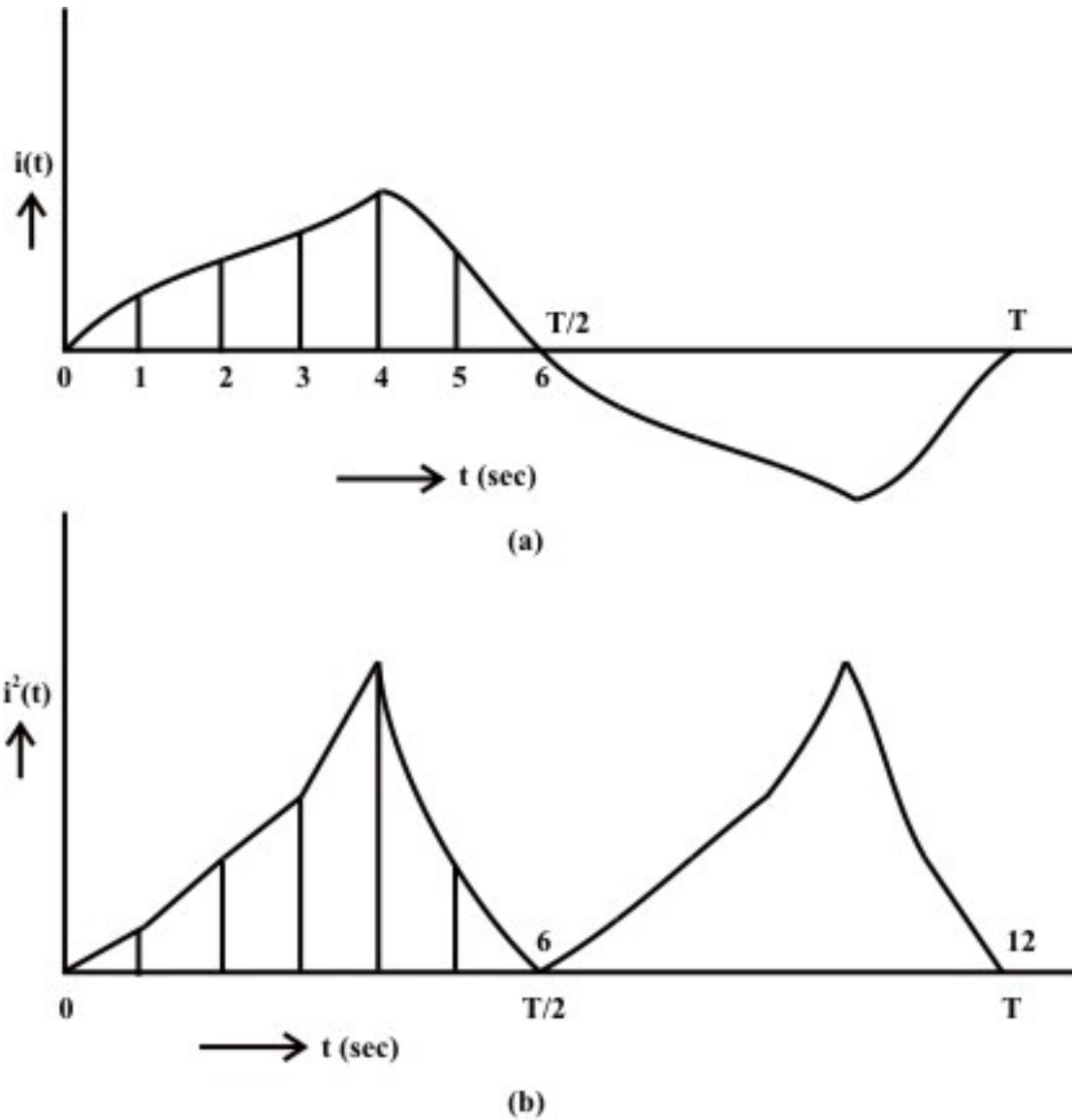
Fig. 12.3 Periodic current waveform
(a) Current (i), (b) Square of current (i²)

The graph of the square of the current waveform, $i^2(t)$ is shown in Fig. 12.3b. Let I be the value of the direct current that produces the same energy dissipated in the resistance R, as produced by the periodic waveform with half time period subdivided into n time intervals,

$$I^2 R = \left[\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2)\, \Delta T}{n \cdot \Delta T} \right] R$$

$$I = \sqrt{\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2)\, \Delta T}{n \cdot \Delta T}} = \sqrt{\frac{Area\ of\ i^2\ curve\ over\ half\ cycle}{Time\ period\ of\ half\ cycle}}$$

$$= \sqrt{\frac{1}{T/2} \int\limits_{0}^{T/2} i^2 \, dt} = \sqrt{\frac{2}{T} \int\limits_{0}^{T/2} i^2 \, dt}$$

This value is termed as Root Mean Square (RMS) or effective one. Also to be noted that the same rms value of the current is obtained using the full cycle, or the time period.

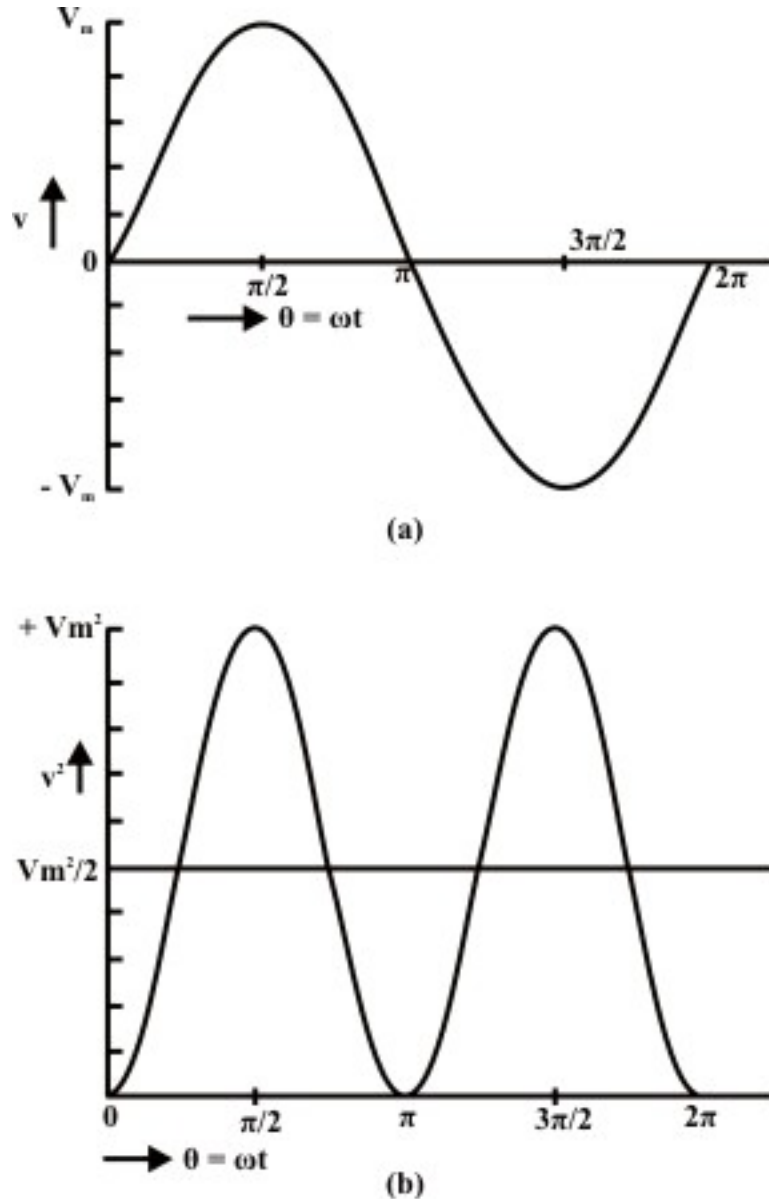## Average and RMS Values of Sinusoidal Voltage Waveform

(a)

(b)

Fig. 12.4 Sinusoidal voltage waveform
(a) Voltage (v), (b) Square of voltage (v²)

As shown earlier, normally the voltage generated, which is also transmitted and then distributed to the consumer, is the sinusoidal waveform with a frequency of 50 Hz in

this country. The waveform of the voltage $v(t)$, and the square of waveform, $v^2(t)$, are shown in figures 12.4a and12.4b respectively.

Time period, $T = 1/f = (2\pi)/\omega$; in angle ($\omega T = 2\pi$)

Half time period, $T/2 = 1/(2f) = \pi/\omega$; in angle ($\omega T/2 = \pi$)

$v(\theta) = V_m \sin\theta \quad for \quad \pi \le \theta \le 0; \quad v(t) = V_m \sin\omega t \quad for \quad (\pi/\omega) \le t \le 0$

$$V_{av} = \frac{1}{\pi}\int_0^\pi v(\theta)\, d\theta = \frac{1}{\pi}\int_0^\pi V_m \sin\theta\, d\theta = \frac{V_m}{\pi}\cos\theta\Big|_\pi^0 = \frac{2}{\pi}V_m = 0.637\, V_m$$

$$V = \left[\frac{1}{\pi}\int_0^\pi v^2\, d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_0^\pi V_m^2 \sin^2\theta\, d\theta\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi}\int_0^\pi \frac{1}{2}(1-\cos 2\theta)\, d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi}(\theta - \frac{1}{2}\sin 2\theta)\Big|_0^\pi\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi}\pi\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

or, $V_m = \sqrt{2}\, V$

If time t, is used as a variable, instead of angle θ,

$$V_{av} = \frac{1}{\pi/\omega}\int_0^{\pi/\omega} v(t)\, dt = \frac{\omega}{\pi}\int_0^{\pi/\omega} V_m \sin\omega t\, dt = \frac{\omega V_m}{\pi\omega}\cos(\omega t)\Big|_\pi^0 = \frac{2}{\pi}V_m = 0.637\, V_m$$

In the same way, the rms value, V can be determined.

If the average value of the above waveform is computed over total time period T, it comes out as <u>zero</u>, as the area of first (positive) half cycle is the same as that of second (negative) half cycle. However, the rms value remains <u>same</u>, if it is computed over total time period.

The different factors are defined as:

Form factor $= \dfrac{RMS\ value}{Average\ value} = \dfrac{0.707\ V_m}{0.637\ V_m} = 1.11$

Peak factor $= \dfrac{Maximum\ value}{Average\ value} = \dfrac{V_m}{0.707\ V_m} = 1.414$

Note: The rms value is always greater than the average value, except for a rectangular waveform, in which case the heating effect remains constant, so that the average and the rms values are same.

Example

The examples of the two waveforms given are periodic in nature.

1. Triangular current waveform (Fig. 12.5)

Time period = T

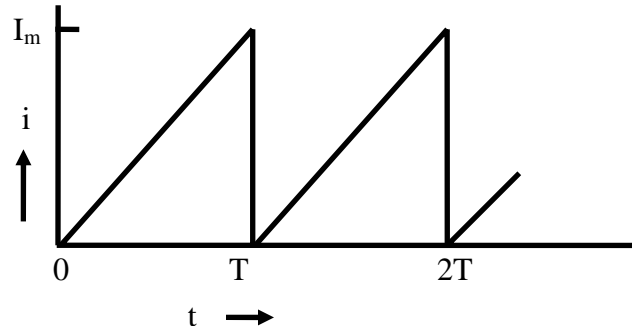$$i(t) = I_m \frac{t}{T} \quad for \quad T \le t \le 0$$

Fig. 12.5 Triangular current waveform

$$I_{av} = \frac{1}{T}\int_0^T i(t)\, dt = \frac{1}{T}\int_0^T I_m\, \frac{t}{T}\, dt = \frac{I_m}{T^2}\, \frac{t^2}{2}\bigg|_0^T = \frac{I_m}{T^2}\, \frac{T^2}{2} = \frac{I_m}{2} = 0.5\, I_m$$

$$I = \left[\frac{1}{T}\int_0^T i^2\, dt\right]^{\frac{1}{2}} = \left[\frac{1}{T}\int_0^T I_m^2\, \frac{t^2}{T^2}\, dt\right]^{\frac{1}{2}} = \left[\frac{I_m^2}{T^3}\cdot\frac{t^3}{3}\bigg|_0^T\right]^{\frac{1}{2}} = \left[\frac{I_m^2}{T^3}\cdot\frac{T^3}{3}\right]^{\frac{1}{2}} = \left[\frac{I_m^2}{3}\right]^{\frac{1}{2}}$$

$$= \frac{I_m}{\sqrt{3}} = 0.57735\, I_m$$

Two factors of the waveform are:

Form factor $= \dfrac{RMS\ value}{Average\ value} = \dfrac{0.57735\, I_m}{0.5\, I_m} = 1.1547$

Peak factor $= \dfrac{Maximum\ value}{Average\ value} = \dfrac{I_m}{0.5\, I_m} = 2.0$

To note that the form factor is slightly higher than that for the sinusoidal waveform, while the peak factor is much higher.
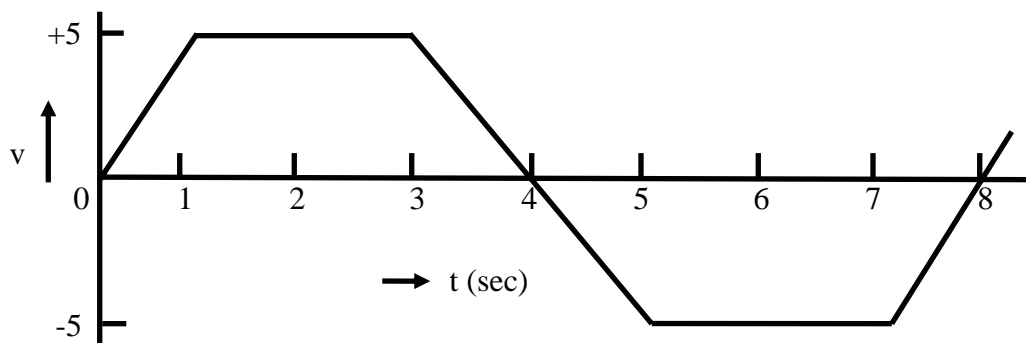

Fig. 12.6 Trapezoidal voltage waveform

2. Trapezoidal voltage waveform (Fig. 12.6)

Time period (T) = 8 ms
Half time period $(T/2) = 8/2 = 4\ ms$

$v(t) = mt = (5/1)t = 5t \ \ for \ \ 1 \le t \le 0 ; \qquad v(t) = 5 \ \ for \ \ 3 \le t \le 1 ;$

$v(t) = 5(4-t) \ \ for \ \ 4 \le t \le 3$

Please note that time, t is in ms, and slope, m is in V/ms. Also to be noted that, as in the case of sinusoidal waveform, only half time period is taken here for the computation of the average and rms values.

$$V_{av} = \frac{1}{T/2} \int_0^{T/2} v(t) \, dt = \frac{1}{4}\left[\int_0^1 5t \, dt + \int_1^3 5 \, dt + \int_3^4 5\,(4-t)\, dt \right] = \frac{1}{4}\left[\frac{5}{2}t^2\Big|_0^1 + 5t\Big|_1^3 + \frac{5}{2}(4-t)^2\Big|_4^3 \right]$$

$$= \frac{1}{4}\left[\frac{5}{2} + 5(3-1) + \frac{5}{2} \right] = \frac{15}{4} = 3.75 \ V$$

$$V = \left[\frac{1}{T/2} \int_0^{T/2} v^2 \, dt \right]^{\frac{1}{2}} = \left[\frac{1}{4}\left(\int_0^1 (5t)^2 \, dt + \int_1^3 (5)^2 \, dt + \int_3^4 5\,(4-t)^2 \, dt \right) \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{4}\left(\frac{25}{3}t^3\Big|_0^1 + 25t\Big|_1^3 + \frac{25}{3}(4-t)^3\Big|_4^3 \right) \right]^{\frac{1}{2}} = \left[\frac{1}{4}\left(\frac{25}{3} + 25\,(3-1) + \frac{25}{3} \right) \right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{50}{3}} = \sqrt{16.67} = 4.0825 \ V$$

Two factors of the waveform are:

$$Form \ factor \ = \frac{RMS \ value}{Average \ value} = \frac{4.0825}{3.75} = 1.0887$$

$$Peak \ factor \ = \frac{Maximum \ value}{Average \ value} = \frac{5.0}{3.75} = 1.3333$$

To note that the both the above factors are slightly lower than those for the sinusoidal waveform.

Similarly, the average and rms or effective values of periodic voltage or current waveforms can be computed.

In this lesson, starting with the generation of single phase ac voltage, the terms, such as average and rms values, related to periodic voltage and current waveforms are explained with examples. In the next lesson, the background material required – the representation of sinusoidal voltage/current as phasors, the rectangular and polar forms of the phasors, as complex quantity, and the mathematical operations – addition/subtraction and multiplication/division, using phasors as complex quantity, are discussed in detail with numerical examples. In the following lessons, the study of circuits fed from single phase ac supply, is presented.

Problems

12.1   What is the speed in rpm of an ac generator with 4 poles, to produce a voltage
       with a frequency of 50 Hz

       (a) 3000   (b) 1500   (c) 1000   (d) 750

12.2   Determine the No. of poles required in an ac generator running at 1,000 rpm, to
       produce a voltage with a frequency of 50 Hz.

       (a) 2   (b) 4   (c) 6   (d) 8

12.3   Calculate the speed in rpm of an ac generator with 24 poles, to produce a voltage
       with a frequency of 50 Hz.

       (a) 300   (b) 250   (c) 200   (d) 150

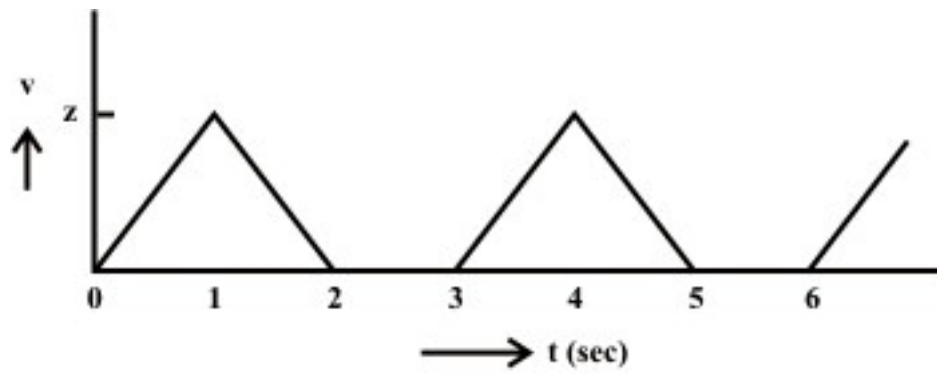12.4   Determine the average and root mean square (rms) values of the following
       waveforms.

(a)
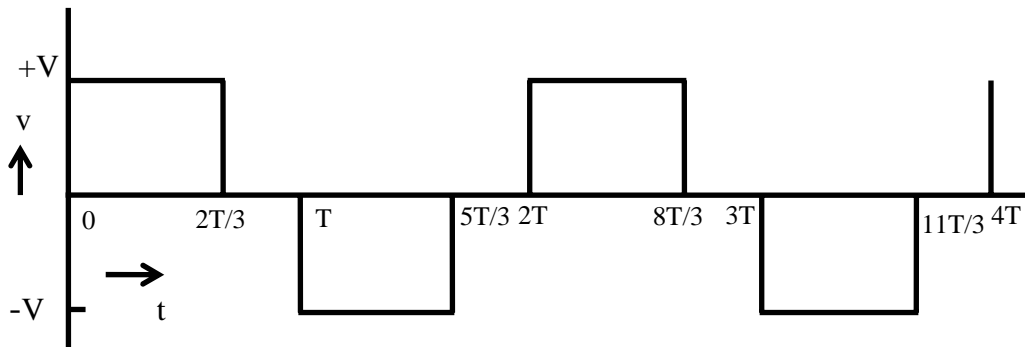
(b)
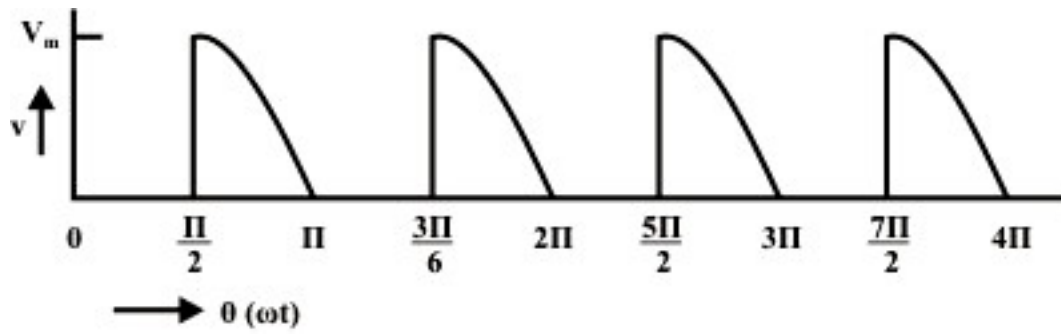
(c)

(d)

Fig. 12.7

List of Figures

Module
4

Single-phase AC Circuits

Lesson
13

Representation of Sinusoidal Signal by a Phasor and Solution of Current in R-L-C Series Circuits

In the last lesson, two points were described:

1. How a sinusoidal voltage waveform (ac) is generated?

2. How the average and rms values of the periodic voltage or current waveforms, are computed?

Some examples are also described there. In this lesson, the representation of sinusoidal (ac) voltage/current signals by a phasor is first explained. The polar/Cartesian (rectangular) form of phasor, as complex quantity, is described. Lastly, the algebra, involving the phasors (voltage/current), is presented. Different mathematical operations – addition/subtraction and multiplication/division, on two or more phasors, are discussed.

Keywords: Phasor, Sinusoidal signals, phasor algebra

After going through this lesson, the students will be able to answer the following questions;

1. What is meant by the term, 'phasor' in respect of a sinusoidal signal?

2. How to represent the sinusoidal voltage or current waveform by phasor?

3. How to write a phasor quantity (complex) in polar/Cartesian (rectangular) form?

4. How to perform the operations, like addition/subtraction and multiplication/division on two or more phasors, to obtain a phasor?

This lesson forms the background of the following lessons in the complete module of single ac circuits, starting with the next lesson on the solution of the current in the steady state, in R-L-C series circuits.

Symbols

i or i(t) Instantaneous value of the current (sinusoidal form)

I Current (rms value)

I_m Maximum value of the current

$\bar{I}$ Phasor representation of the current

ϕ Phase angle, say of the current phasor, with respect to the reference phasor

Same symbols are used for voltage or any other phasor.

Representation of Sinusoidal Signal by a Phasor

A sinusoidal quantity, i.e. current, $i(t) = I_m \sin \omega t$, is taken up as an example. In Fig. 13.1a, the length, OP, along the x-axis, represents the maximum value of the current I_m, on a certain scale. It is being rotated in the anti-clockwise direction at an angular speed, ω, and takes up a position, OA after a time t (or angle, $\theta = \omega t$, with the x-axis). The vertical projection of OA is plotted in the right hand side of the above figure with respect to the angle θ. It will generate a sine wave (Fig. 13.1b), as OA is at an angle, θ with the x-axis, as stated earlier. The vertical projection of OA along y-axis is OC = AB =

$i(\theta) = I_m \sin\theta$, which is the instantaneous value of the current at any time t or angle θ. The angle θ is in rad., i.e. $\theta = \omega t$. The angular speed, ω is in rad/s, i.e. $\omega = 2\pi f$, where f is the frequency in Hz or cycles/sec. Thus,

$$i = I_m \sin\theta = I_m \sin\omega t = I_m \sin 2\pi ft$$

So, OP represents the phasor with respect to the above current, i.

The line, OP can be taken as the rms value, $I = I_m / \sqrt{2}$, instead of maximum value, I_m. Then the vertical projection of OA, in magnitude equal to OP, does not represent exactly the instantaneous value of I, but represents it with the scale factor of $1/\sqrt{2} = 0.707$. The reason for this choice of phasor as given above, will be given in another lesson later in this module.
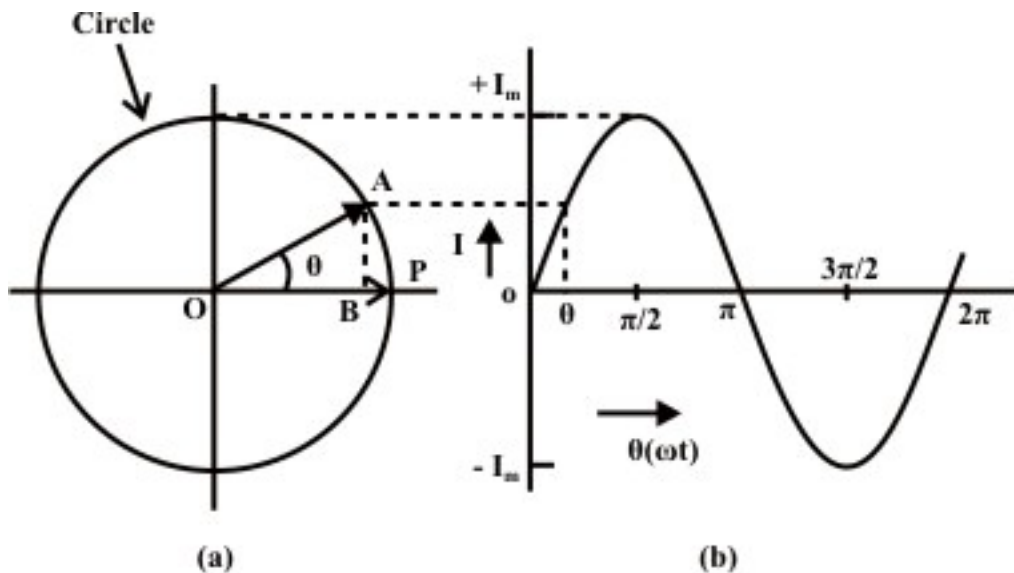

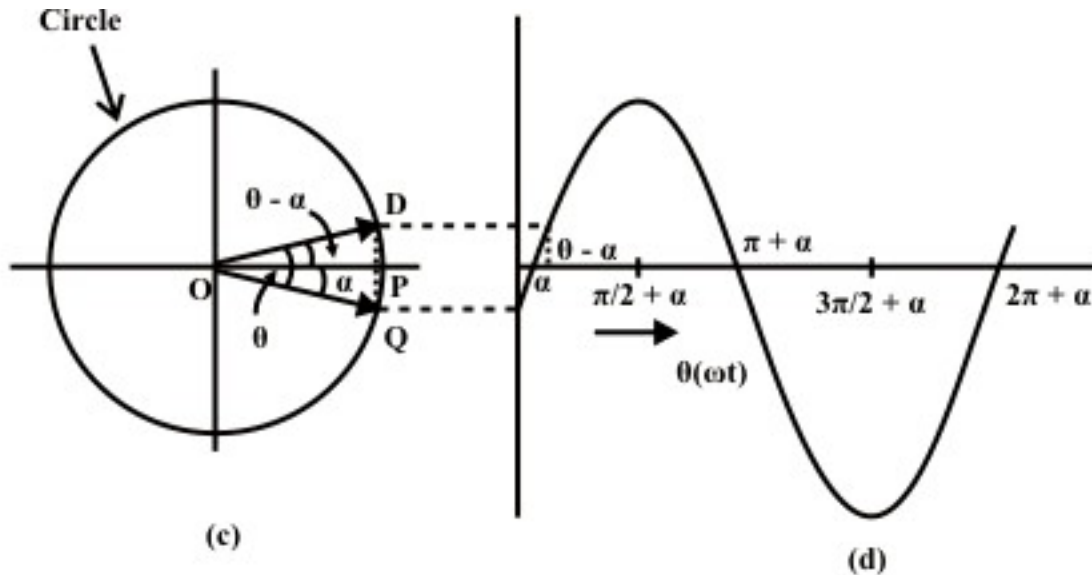Fig. 13.1(a) Phasor representation of a sinusoidal current, and (b) Waveform

Fig. 13.1 (c) Phasor representation of a phase shifted sinusoidal current, and (d) Waveform

Generalized case

The current can be of the form, $i(t) = I_m \sin(\omega t - \alpha)$ as shown in Fig. 13.1d. The phasor representation of this current is the line, OQ, at an angle, α (may be taken as negative), with the line, OP along x-axis (Fig. 13.1c). One has to move in clockwise direction to go to OQ from OP (reference line), though the phasor, OQ is assumed to move in anti-clockwise direction as given earlier. After a time t, OD will be at an angle θ with OQ, which is at an angle $(\theta - \alpha = \omega t - \alpha)$, with the line, OP along x-axis. The vertical projection of OD along y-axis gives the instantaneous value of the current,

$$i = \sqrt{2} I \sin(\omega t - \alpha) = I_m \sin(\omega t - \alpha).$$

Phasor representation of Voltage and Current

The voltage and current waveforms are given as,

$$v = \sqrt{2} V \sin\theta, \text{ and } i = \sqrt{2} I \sin(\theta + \phi)$$

It can be seen from the waveforms (Fig. 13.2b) of the two sinusoidal quantities – voltage and current, that the voltage, V lags the current I, which means that the positive maximum value of the voltage is reached earlier by an angle, ϕ, as compared to the positive maximum value of the current. In phasor notation as described earlier, the voltage and current are represented by OP and OQ (Fig. 13.2a) respectively, the length of which are proportional to voltage, V and current, I in different scales as applicable to each one. The voltage phasor, OP (V) lags the current phasor, OQ (I) by the angle ϕ, as two phasors rotate in the anticlockwise direction as stated earlier, whereas the angle ϕ is also measured in the anticlockwise direction. In other words, the current phasor (I) leads the voltage phasor (V).
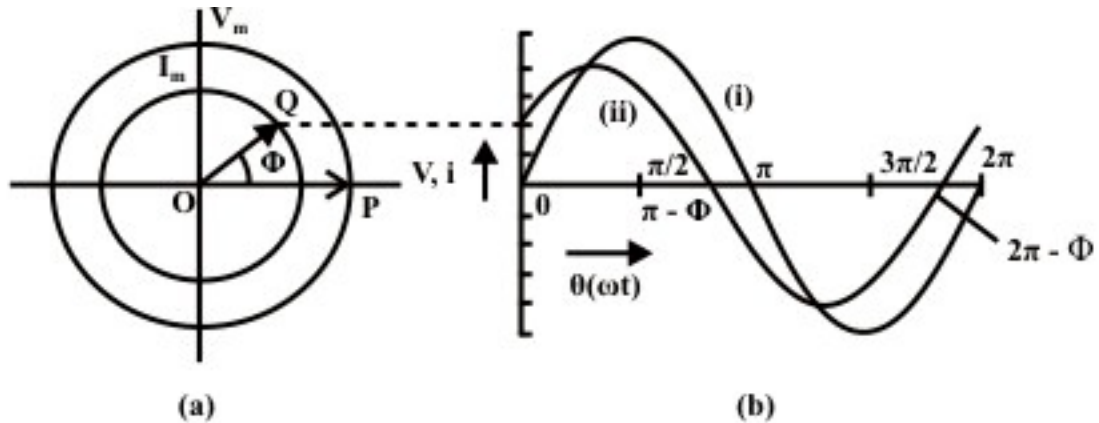
Fig. 13.2 (a) Phasor representation of a sinusoidal (i) voltage and (ii) current, and (b) Waveforms

Mathematically, the two phasors can be represented in polar form, with the voltage phasor ($\bar{V}$) taken as reference, such as $\bar{V} = V \angle 0^0$, and $\bar{I} = I \angle \phi$.

In Cartesian or rectangular form, these are,

$\bar{V} = V \angle 0^0 = V + j0$, and $\bar{I} = I \angle \phi = I \cos \phi + j I \sin \phi$,

where, the symbol, j is given by $j = \sqrt{-1}$.

Of the two terms in each phasor, the first one is termed as real or its component in x-axis, while the second one is imaginary or its component in y-axis, as shown in Fig. 13.3a. The angle, ϕ is in degree or rad.

Phasor Algebra

Before discussing the mathematical operations, like addition/subtraction and multi-plication/division, involving phasors and also complex quantities, let us take a look at the two forms – polar and rectangular, by which a phasor or complex quantity is represented. It may be observed here that phasors are also taken as complex, as given above.
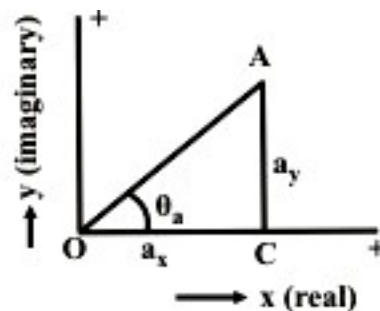

Fig. 13.3 Representation of a phasor, both in rectangular and polar forms

Representation of a phasor and Transformation

A phasor or a complex quantity in rectangular form (Fig. 13.3) is,

$\bar{A} = a_x + j a_y$

where a_x and a_y are real and imaginary parts, of the phasor respectively.

In polar form, it is expressed as

$$\bar{A} = A \angle \theta_a = A \cos \theta_a + j A \sin \theta_a$$

where A and θ_a are magnitude and phase angle of the phasor.

From the two equations or expressions, the procedure or rule of transformation from polar to rectangular form is

$$a_x = A \cos \theta_a \text{ and } a_y = A \sin \theta_a$$

From the above, the rule for transformation from rectangular to polar form is

$$A = \sqrt{a_x^2 + a_y^2} \text{ and } \theta_a = \tan^{-1}\left(a_y / a_x\right)$$

The examples using numerical values are given at the end of this lesson.

Addition/Subtraction of Phasors

Before describing the rules of addition/subtraction of phasors or complex quantities, everyone should recall the rule of addition/subtraction of scalar quantities, which may be positive or signed (decimal/fraction or fraction with integer). It may be stated that, for the two operations, the quantities must be either phasors, or complex. The example of phasor is voltage/current, and that of complex quantity is impedance/admittance, which will be explained in the next lesson. But one phasor and another complex quantity should not be used for addition/subtraction operation.

For the operations, the two phasors or complex quantities must be expressed in rectangular form as

$$\bar{A} = a_x + j a_y ; \quad \bar{B} = b_x + j b_y$$

If they are in polar form as

$$\bar{A} = A \angle \theta_a ; \quad \bar{B} = B \angle \theta_b$$

In this case, two phasors are to be transformed to rectangular form by the procedure or rule given earlier.

The rule of addition/subtraction operation is that both the real and imaginary parts have to be separately treated as

$$\bar{C} = \bar{A} \pm \bar{B} = \left(a_x \pm b_x\right) + j\left(a_y \pm b_y\right) = c_x + j c_y$$

where $c_x = \left(a_x \pm b_x\right) ; \quad c_y = \left(a_y \pm b_y\right)$

Say, for addition, real parts must be added, so also for imaginary parts. Same rule follows for subtraction. After the result is obtained in rectangular form, it can be transformed to polar one. It may be observed that the six values of $a's$, $b's$ and $c's$ – parts of the two phasors and the resultant one, are all signed scalar quantities, though in the example, $a's$ and $b's$ are taken as positive, resulting in positive values of $c's$. Also the phase angle $\theta's$ may lie in any of the four quadrants, though here the angles are in the first quadrant only.

This rule for <u>addition</u> can be extended to three or more quantities, as will be illustrated through example, which is given at the end of this lesson.
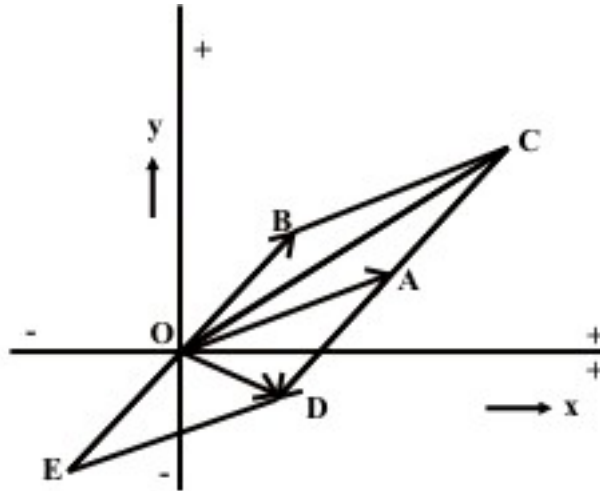
Fig. 13.4 Addition and subtraction of two phasors, both represented in polar form

The addition/subtraction operations can also be performed using the quantities as phasors in polar form (Fig. 13.4). The two phasors are $\bar{A}(OA)$ and $\bar{B}(OB)$. The find the sum $\bar{C}(OC)$, a line AC is drawn equal and parallel to OB. The line BC is equal and parallel to OA. Thus, $\bar{C} = OC = OA + AC = OA + OB = \bar{A} + \bar{B}$. Also,

$$OC = OB + BC = OB + OA$$

To obtain the difference $\bar{D}(OD)$, a line AD is drawn equal and parallel to OB, but in opposite direction to AC or OB. A line OE is also drawn equal to OB, but in opposite direction to OB. Both AD and OE represent the phasor $(-\bar{B})$. The line, ED is equal to OA. Thus, $\bar{D} = OD = OA + AD = OA - OB = \bar{A} - \bar{B}$. Also $OD = OE + ED = -OB + OA$. The examples using numerical values are given at the end of this lesson.

Multiplication/Division of Phasors

Firstly, the procedure for multiplication is taken up. In this case no reference is being made to the rule involving scalar quantities, as everyone is familiar with them. Assuming that the two phasors are available in polar from as $\bar{A} = A\angle\theta_a$ and $\bar{B} = B\angle\theta_b$. Otherwise, they are to be transformed from rectangular to polar form. This is also valid for the procedure of division. Please note that a phasor is to be multiplied by a complex quantity only, to obtain the resultant phasor. A phasor is not normally multiplied by another phasor, except in special case. Same is for division. A phasor is to be divided by a complex quantity only, to obtain the resultant phasor. A phasor is not normally divided by another phasor.

To find the magnitude of the product $\bar{C}$, the two magnitudes of the phasors are to be multiplied, whereas for phase angle, the phase angles are to added. Thus,

$$\bar{C} = C \angle \theta_c = \bar{A} \cdot \bar{B} = A \angle \theta_A \cdot B \angle \theta_B = (A \cdot B) \angle (\theta_a + \theta_b)$$

where $C = A \cdot B$ and $\theta_c = \theta_a + \theta_b$

Please note that the same symbol, $\bar{C}$ is used for the product in this case.

To divide $\bar{A}$ .by $\bar{B}$ to obtain the result $\bar{D}$., the magnitude is obtained by division of the magnitudes, and the phase is difference of the two phase angles. Thus,

$$\bar{D} = D \angle \theta_d = \frac{\bar{A}}{\bar{B}} = \frac{A \angle \theta_a}{B \angle \theta_b} = \left(\frac{A}{B}\right) \angle (\theta_a - \theta_b)$$

where $D = A / B$ and $\theta_d = \theta_a - \theta_b$

If the phasors are expressed in rectangular form as

$\bar{A} = a_x + j a_y$ and $\bar{B} = b_x + j b_y$

where $A = \sqrt{(a_x^2 + a_y^2)}$; $\theta_a = \tan^{-1}(a_y / a_x)$

The values of $\bar{B}$ are not given as they can be obtained by substituting $b's$ for $a's$. To find the product,

$$\bar{C} = C \angle \theta_c = \bar{A} \cdot \bar{B} = (a_x + j a_y) \cdot (b_x + j b_y) = (a_x b_x - a_y b_y) + j(a_x b_y + a_y b_x)$$

Please note that $j^2 = -1$ .The magnitude and phase angle of the result (phasor) are,

$$C = \left[(a_x b_x - a_y b_y)^2 + (a_x b_y + a_y b_x)^2\right]^{\frac{1}{2}} = \sqrt{(a_x^2 + a_y^2)} \cdot \sqrt{(b_x^2 + b_y^2)} = A \cdot B \text{ , and}$$

$$\theta_c = \tan^{-1}\left(\frac{a_x b_y + a_y b_x}{a_x b_x - a_y b_y}\right)$$

The phase angle,

$$\theta_c = \theta_a + \theta_b = \tan^{-1}\left(\frac{a_y}{a_x}\right) + \tan^{-1}\left(\frac{b_y}{b_x}\right) = \tan^{-1}\left[\frac{(a_y / a_x) + (b_y / b_x)}{1 - (a_y / a_x) \cdot (b_y / b_x)}\right]$$

$$= \tan^{-1}\left(\frac{a_x b_y + a_y b_x}{a_x b_x - a_y b_y}\right)$$

The above results are obtained by simplification.

To divide $\bar{A}$ by $\bar{B}$ to obtain $\bar{D}$ as

$$\bar{D} = d_x + j d_y = \frac{\bar{A}}{\bar{B}} = \frac{a_x + j a_y}{b_x + j b_y}$$

To simplify $\bar{D}$, i.e. to obtain real and imaginary parts, both numerator and denominator, are to be multiplied by the complex conjugate of $\bar{B}$, so as to convert the denominator into real value only. The complex conjugate of $\bar{B}$ is

$B^* = b_x + jb_y = B\angle -\theta_b$

In the complex conjugate, the sign of the imaginary part is negative, and also the phase angle is negative.

$$\bar{D} = d_x + jd_y = \frac{(a_x + ja_y)\cdot(b_x - jb_y)}{(b_x + jb_y)\cdot(b_x - jb_y)} = \left(\frac{a_xb_x + a_yb_y}{b_x^2 + b_y^2}\right) + j\left(\frac{a_yb_x - a_xb_y}{b_x^2 + b_y^2}\right)$$

The magnitude and phase angle of the result (phasor) are,

$$D = \frac{\left[(a_xb_x + a_yb_y)^2 + (a_yb_x - a_xb_y)^2\right]^{\frac{1}{2}}}{(b_x^2 + b_y^2)} = \frac{\sqrt{(a_x^2 + a_y^2)}}{\sqrt{(b_x^2 + b_y^2)}} = \frac{A}{B} \text{, and}$$

$$\theta_d = \tan^{-1}\left(\frac{a_yb_x - a_xb_y}{a_xb_x + a_yb_y}\right)$$

The phase angle,

$$\theta_d = \theta_a - \theta_b = \tan^{-1}\left(\frac{a_y}{a_x}\right) - \tan^{-1}\left(\frac{b_y}{b_x}\right) = \tan^{-1}\left(\frac{a_yb_x - a_xb_y}{a_xb_x + a_yb_y}\right)$$

The steps are shown here in brief, as detailed steps have been given earlier.
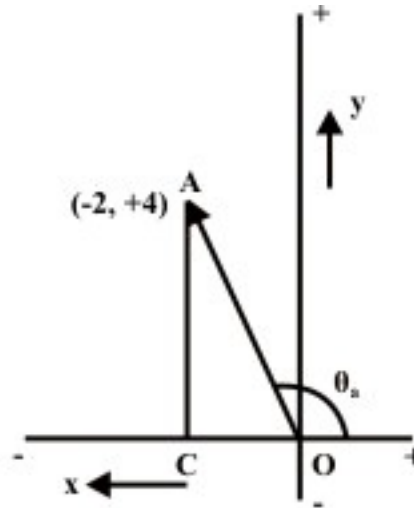
Example

Fig. 13.5 Representation of phasor as an example, both in rectangular and polar forms

The phasor, $\bar{A}$ in the rectangular form (Fig. 13.5) is,

$$\bar{A} = A\angle\theta_a = A\cos\theta_a + jA\sin\theta_a = a_x + ja_y = -2 + j4$$

where the real and imaginary parts are $a_x = -2$; $a_y = 4$

To transform the phasor, $\bar{A}$ into the polar form, the magnitude and phase angle are

$$A = \sqrt{a_x^2 + a_y^2} = \sqrt{(-2)^2 + 4^2} = 4.472$$

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{4}{-2}\right) = 116.565° = 2.034\,rad$$

Please note that θ_a is in the second quadrant, as real part is negative and imaginary part is positive.

Transforming the phasor, $\bar{A}$ into rectangular form, the real and imaginary parts are

$$a_x = A\cos\theta_a = 4.472 \cdot \cos116.565° = -2.0$$
$$a_y = A\sin\theta_a = 4.472 \cdot \sin116.565° = 4.0$$

Phasor Algebra
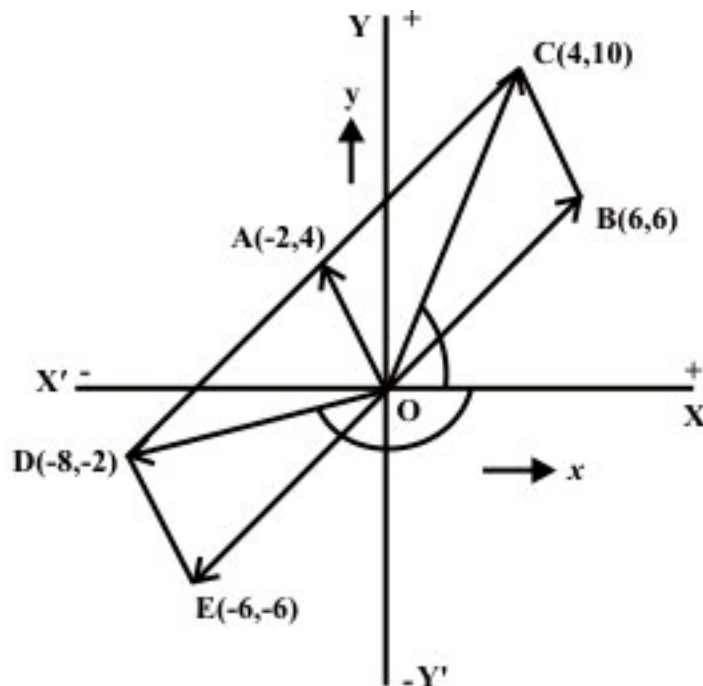
Fig.13.6 Addition and subtraction of two phasors represented in polar form, as an example

Another phasor, $\bar{B}$ in rectangular form is introduced in addition to the earlier one, $\bar{A}$

$$\bar{B} = 6 + j6 = 8.485\angle45°$$

Firstly, let us take the addition and subtraction of the above two phasors. The sum and difference are given by the phasors, $\bar{C}$ and $\bar{D}$ respectively (Fig. 13.6).

$$\bar{C} = \bar{A} + \bar{B} = (-2 + j4) + (6 + j6) = (-2 + 6) + j(4 + 6) = 4 + j10 = 10.77\angle68.2°$$

$$\bar{D} = \bar{A} - \bar{B} = (-2 + j4) - (6 + j6) = (-2 - 6) + j(4 - 6) = -8 - j2 = 8.246\angle-166.0°$$

It may be noted that for the addition and subtraction operations involving phasors, they should be represented in rectangular form as given above. If any one of the phasors

is in polar form, it should be transformed into rectangular form, for calculating the results as shown.

If the two phasors are both in polar form, the phasor diagram (the diagram must be drawn to scale), or the geometrical method can be used as shown in Fig 13.6. The result obtained using the diagram, as shown are the same as obtained earlier.

$[\bar{C}\,(\text{OC}) = 10.77,\ \angle COX = 68.2°\,;\ \text{and}\ \bar{D}\,(\text{OD}) = 8.246,\ \angle DOX = 166.0°]$

Now, the multiplication and division operations are performed, using the above two phasors represented in polar form. If any one of the phasors is in rectangular form, it may be transformed into polar form. Also note that the same symbols for the phasors are used here, as was used earlier. Later, the method of both multiplication and division using rectangular form of the phasor representation will be explained.

The resultant phasor $\bar{C}$, i.e. the product of the two phasors is

$$\bar{C} = \bar{A}\cdot\bar{B} = 4.472\angle116.565°\times8.485\angle45° = (4.472\times8.485)\angle(116.565°+45°)$$
$$= 37.945\angle161.565° = -36 + j12$$

The product of the two phasors in rectangular form can be found as

$$\bar{C} = (-2 + j4)\cdot(6 + j6) = (-12 - 24) + j(24 - 12) = -36 + j12$$

The result ($\bar{D}$) obtained by the division of $\bar{A}$ by $\bar{B}$ is

$$\bar{D} = \frac{\bar{A}}{\bar{B}} = \frac{4.472\angle116.565°}{8.485\angle45°} = \left(\frac{4.472}{8.485}\right)\angle(116.565° - 45°) = 0.527\angle71.565°$$
$$= 0.167 + j0.5$$

The above result can be calculated by the procedure described earlier, using the rectangular form of the two phasors as

$$\bar{D} = \frac{\bar{A}}{\bar{B}} = \frac{-2 + j4}{6 + j6} = \frac{(-2 + j4)\cdot(6 - j6)}{(6 + j6)\cdot(6 - j6)} = \frac{(-12 + 24) + j(24 + 12)}{6^2 + 6^2}$$
$$= \frac{12 + j36}{72} = 0.167 + j0.5$$

The procedure for the elementary operations using two phasors only, in both forms of representation is shown. It can be easily extended, for say, addition/multiplication, using three or more phasors. The simplification procedure with the scalar quantities, using the different elementary operations, which is well known, can be extended to the phasor quantities. This will be used in the study of ac circuits to be discussed in the following lessons.

The background required, i.e. phasor representation of sinusoidal quantities (voltage/current), and algebra – mathematical operations, such as addition/subtraction and multiplication/division of phasors or complex quantities, including transformation of phasor from rectangular to polar form, and vice versa, has been discussed here. The study of ac circuits, starting from series ones, will be described in the next few lessons.

Problems

13.1 Use plasor technique to evaluate the expression and then find the numerical value at t = 10 ms.

$$i(t) = 150\cos\left(100t - 45^0\right) + 500\sin\left(100t\right) + \frac{d}{dt}\left[\cos\left(100t - 30^0\right)\right]$$

13.2 Find the result in both rectangular and polar forms, for the following, using complex quantities:

a) $\dfrac{5 - j12}{15\angle 53.1^\circ}$

b) $\left(5 - j12\right) + 15\angle - 53.1^\circ$

c) $\dfrac{2\angle 30^\circ - 4\angle 210^\circ}{5\angle 450^\circ}$

d) $\left(5\angle 0^\circ + \dfrac{1}{3\sqrt{2}\angle - 45^\circ}\right).2\angle 210^\circ$

List of Figures

Module
4

Single-phase AC Circuits

Lesson
14

Solution of Current in R-L-C Series Circuits

In the last lesson, two points were described:

1. How to represent a sinusoidal (ac) quantity, i.e. voltage/current by a phasor?

2. How to perform elementary mathematical operations, like addition/ subtraction and multiplication/division, of two or more phasors, represented as complex quantity?

Some examples are also described there. In this lesson, the solution of the steady state currents in simple circuits, consisting of resistance R, inductance L and/or capacitance C connected in series, fed from single phase ac supply, is presented. Initially, only one of the elements R / L / C, is connected, and the current, both in magnitude and phase, is computed. Then, the computation of total reactance and impedance, and the current, in the circuit consisting of two components, R & L / C only in series, is discussed. The process of drawing complete phasor diagram with current(s) and voltage drops in the different components is described. Lastly, the computation of total power and also power consumed in the components, along with the concept of power factor, is explained.

Keywords: Series circuits, reactance, impedance, phase angle, power, power factor.

After going through this lesson, the students will be able to answer the following questions;

1. How to compute the total reactance and impedance of the R-L-C series circuit, fed from single phase ac supply of known frequency?

2. How to compute the current and also voltage drops in the components, both in magnitude and phase, of the circuit?

3. How to draw the complete phasor diagram, showing the current and voltage drops?

4. How to compute the total power and also power consumed in the components, along with power factor?

Solution of Steady State Current in Circuits Fed from Single-phase AC Supply

Elementary Circuits

1. **Purely resistive circuit (R only)**

The instantaneous value of the current though the circuit (Fig. 14.1a) is given by,

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where,

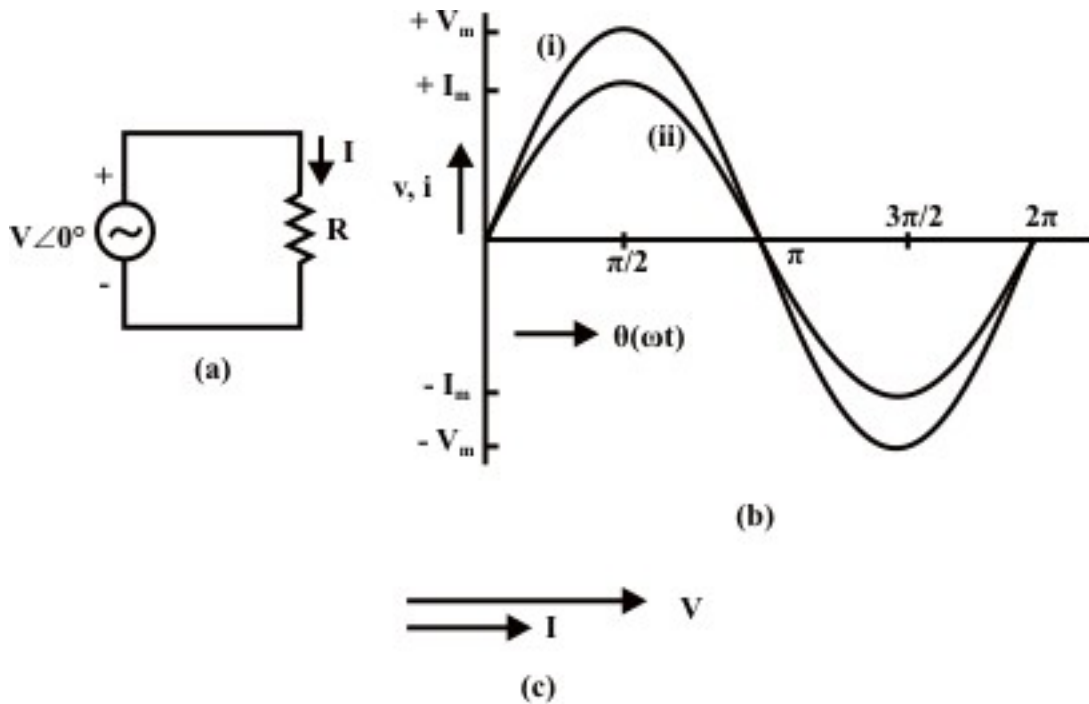I_m and V_m are the maximum values of current and voltage respectively.

Fig. 14.1: Circuit with Resistance (R)
(a) Circuit diagram
(b) Waveforms: (i) Voltage (ii) Current
(c) Phasor diagram

The rms value of current is given by

$$\bar{I} = \frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{R} = \frac{\bar{V}}{R}$$

In phasor notation,

$$\bar{V} = V\angle 0° = V(1 + j0) = V + j0$$

$$\bar{I} = I\angle 0° = I(1 + j0) = I + j0$$

The impedance or resistance of the circuit is obtained as,

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle 0°} = Z\angle 0° = R + j0$$

Please note that the voltage and the current are in phase ($\phi = 0°$), which can be observed from phasor diagram (Fig. 14.1b) with two (voltage and current) phasors, and also from the two waveforms (Fig. 14.1c).

In ac circuit, the term, Impedance is defined as voltage/current, as is the resistance in dc circuit, following Ohm's law. The impedance, Z is a complex quantity. It consists of real part as resistance R, and imaginary part as reactance X, which is zero, as there is no inductance/capacitance. All the components are taken as constant, having linear V-I characteristics. In the three cases being considered, including this one, the power

consumed and also power factor in the circuits, are not taken up now, but will be described later in this lesson.

2. Purely inductive circuit (L only)

For the circuit (Fig. 14.2a), the current i, is obtained by the procedure described here.

As $v = L\dfrac{di}{dt} = V_m \sin \omega t = \sqrt{2} V \sin \omega t$,

$di = \dfrac{\sqrt{2} V}{L} \sin(\omega t) \, dt$

Integrating,

$i = -\dfrac{\sqrt{2} V}{\omega L} \cos \omega t = \dfrac{\sqrt{2} V}{\omega L} \sin(\omega t - 90°) = I_m \sin(\omega t - 90°) = \sqrt{2} I \sin(\omega t - 90°)$
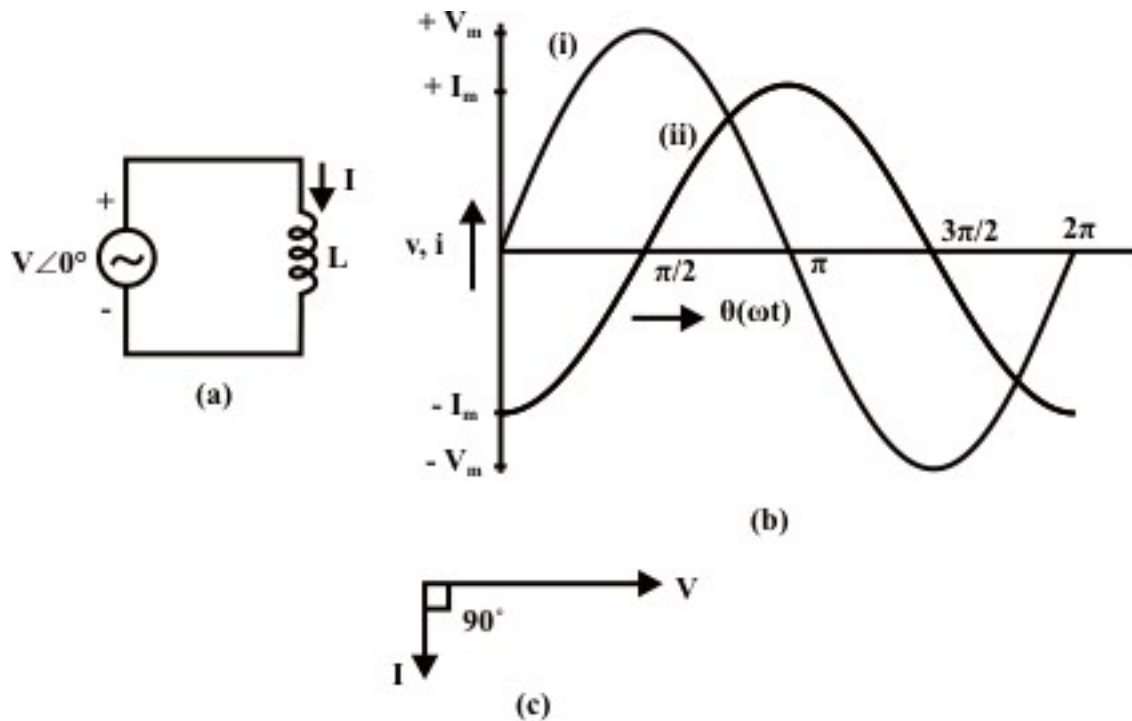
Fig. 14.2: Circuit with Inductance (L)
(a) Circuit diagram
(b) Waveforms: (i) Voltage (ii) Current
(c) Phasor diagram

It may be mentioned here that the current i, is the steady state solution, neglecting the constant of integration. The rms value, I is

$\bar{I} = \dfrac{\bar{V}}{\omega L} = I \angle -90°$

$\bar{V} = V \angle 0° = V + j0 \quad ; \qquad \bar{I} = I \angle -90° = 0 - jI$

The impedance of the circuit is

$$Z \angle \phi = \frac{\bar{V}}{\bar{I}} = \frac{V \angle 0°}{I \angle -90°} = \frac{V}{-jI} = j\omega L = 0 + j X_L = X_L \angle 90° = \omega L \angle 90°$$

where, the inductive reactance is $X_L = \omega L = 2 \pi f L$.

Note that the current lags the voltage by $\phi = +90°$. This can be observed both from phasor diagram (Fig. 14.2b), and waveforms (Fig. 14.2c). As the circuit has no resistance, but only inductive reactance $X_L = \omega L$ (positive, as per convention), the impedance Z is only in the y-axis (imaginary).

3. **Purely capacitive circuit (C only)**

The current i, in the circuit (Fig. 14.3a), is,

$$i = C \frac{dv}{dt}$$

Substituting $v = \sqrt{2} V \sin \omega t = V_m \sin \omega t$, i is

$$i = C \frac{d}{dt}\left(\sqrt{2} V \sin \omega t\right) = \sqrt{2} \omega CV \cos \omega t = \sqrt{2} \omega CV \sin(\omega t + 90°) = \sqrt{2} I \sin(\omega t + 90°)$$

$$= I_m \sin(\omega t + 90°)$$

The rms value, I is

$$\bar{I} = \omega C \bar{V} = \frac{\bar{V}}{1/(\omega C)} = I \angle 90°$$

$$\bar{V} = V \angle 0° = V + j0 \quad ; \qquad \bar{I} = I \angle 90° = 0 + jI$$

The impedance of the circuit is

$$Z \angle \phi = \frac{\bar{V}}{\bar{I}} = \frac{V \angle 0°}{I \angle 90°} = \frac{V}{jI} = \frac{1}{j\omega C} = -\frac{j}{\omega C} = 0 - j X_C = X_C \angle -90° = \frac{1}{\omega C} \angle 90°$$

where, the capacitive reactance is $X_C = \frac{1}{\omega C} = \frac{1}{2 \pi f C}$.

Note that the current leads the voltage by $\phi = 90°$ (this value is negative, i.e. $\phi = -90°$), as per convention being followed here. This can be observed both from phasor diagram (Fig. 14.3b), and waveforms (Fig. 14.3c). As the circuit has no resistance, but only capacitive reactance, $X_c = 1/(\omega C)$ (negative, as per convention), the impedance Z is only in the y-axis (imaginary).
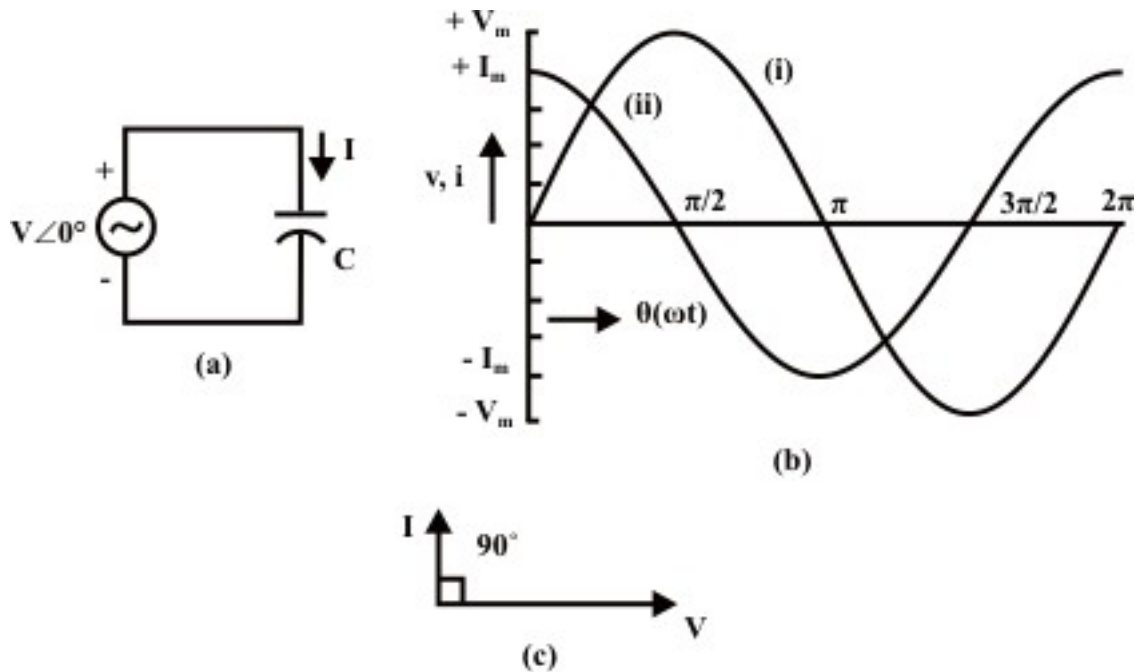
Fig. 14.3: Circuit with Capacitance (C)
(a) Circuit diagram
(b) Waveforms: (i) Voltage (ii) Current
(c) Phasor diagram

Series Circuits

1. **Inductive circuit (R and L in series)**

The voltage balance equation for the R-L series circuit (Fig. 14.4a) is,

$$v = R\,i + L\frac{di}{dt}$$

where, $v = \sqrt{2}\,V \sin \omega t = V_m \sin \omega t = \sqrt{2}\,V \sin \theta$, θ being ωt.

The current, i (in steady state) can be found as

$$i = \sqrt{2}\,I \sin (\omega t - \phi) = I_m \sin (\omega t - \phi) = \sqrt{2}\,I \sin (\theta - \phi)$$

The current, $i(t)$ in steady state is sinusoidal in nature (neglecting transients of the form shown in the earlier module on dc transients). This can also be observed, if one sees the expression of the current, $i = I_m \sin (\omega t)$ for purely resistive case (with R only), and $i = I_m \sin (\omega t - 90°)$ for purely inductive case (with L only).

Alternatively, if the expression for i is substituted in the voltage equation, the equation as given here is obtained.

$$\sqrt{2}\,V \sin \omega t = R \cdot \sqrt{2}\,I \sin (\omega t - \phi) + \omega L \cdot \sqrt{2}\,I \cos (\omega t - \phi)$$

If, first, the trigonometric forms in the RHS side is expanded in terms of $\sin \omega t$ and $\cos \omega t$, and then equating the terms of $\sin \omega t$ and $\cos \omega t$ from two (LHS & RHS) sides, the two equations as given here are obtained.

$$V = (R \cdot \cos \phi + \omega L \cdot \sin \phi) \cdot I \text{ , and}$$

$$0 = (-R \cdot \sin\phi + \omega L \cdot \cos\phi)$$

From these equations, the magnitude and phase angle of the current, I are derived.

From the second one, $\tan\phi = (\omega L / R)$

So, phase angle, $\phi = \tan^{-1}(\omega L / R)$

Two relations, $\cos\phi = (R/Z)$, and $\sin\phi = (\omega L/Z)$, are derived, with the term (impedance), $Z = \sqrt{R^2 + (\omega L)^2}$

If these two expressions are substituted in the first one, it can be shown that the magnitude of the current is $I = V/Z$, with both V and Z in magnitude only.

The steps required to find the rms value of the current I, using complex form of impedance, are given here.
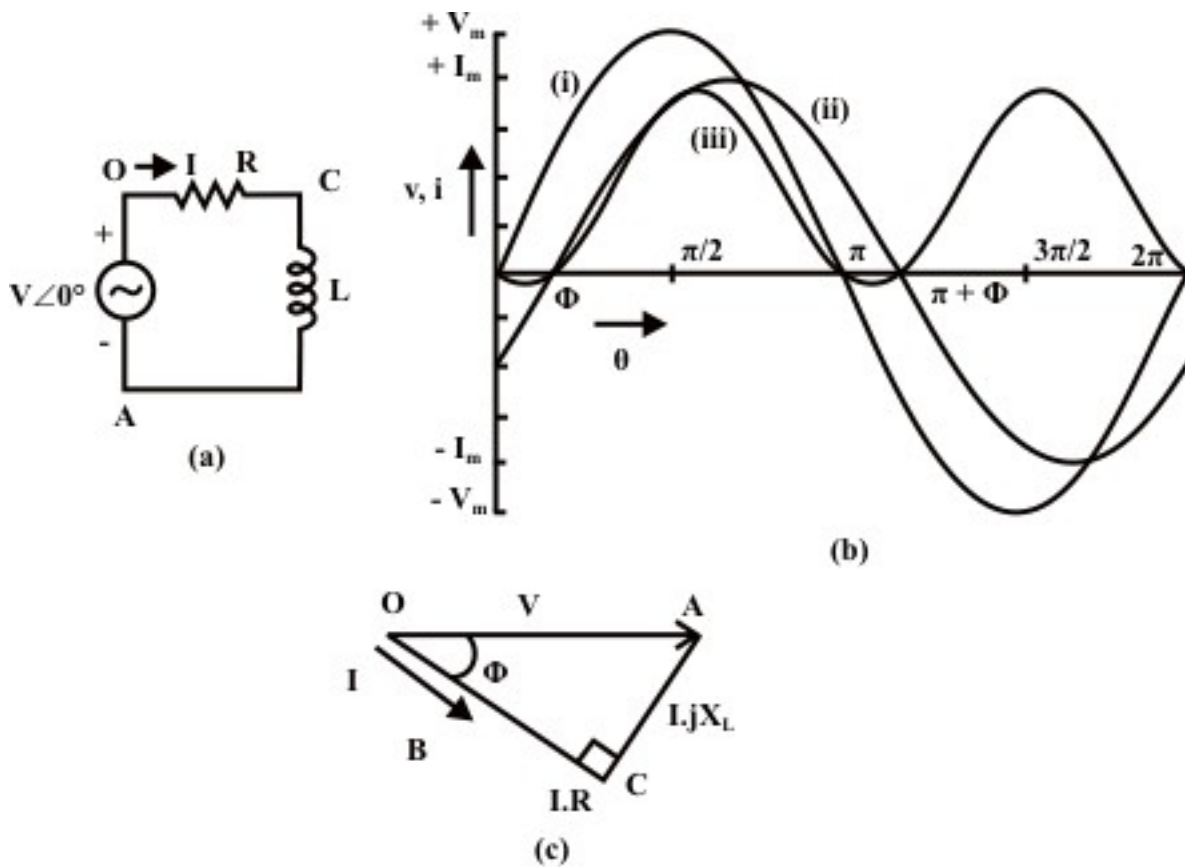
Fig. 14.4: Circuit with Resistance (R) and Inductance (L) in series.
(a) Circuit diagram
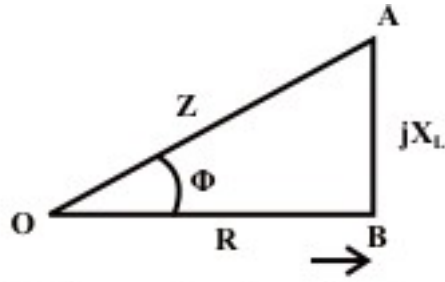(b) Waveforms: (i) Voltage (ii) Current (iii) Power
(c) Phasor diagram

Fig. 14.5: The complex form of the impedance (R-L series circuit)

The impedance (Fig. 14.5) of the inductive (R-L) circuit is,

$$Z\angle\phi = R + j X_L = R + j\omega L$$

where,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\bar{I}\angle -\phi = \frac{V\angle 0°}{Z\angle\phi} = \frac{V + j0}{R + j X_L} = \frac{V + j0}{R + j\omega L}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{\sqrt{R^2 + (\omega L)^2}}$$

Note that the current lags the voltage by the angle ϕ, value as given above. In this case, the voltage phasor has been taken as reference phase, with the current phasor lagging the voltage phasor by the angle, ϕ. But normally, in the case of the series circuit, the current phasor is taken as reference phase, with the voltage phasor leading the current phasor by ϕ. This can be observed both from phasor diagram (Fig. 14.4b), and waveforms (Fig. 14.4c). The inductive reactance X_L is positive. In the phasor diagram, as one move from voltage phasor to current phasor, one has to go in the <u>clockwise</u> direction, which means that phase angle, ϕ is taken as positive, though both phasors are assumed to move in anticlockwise direction as shown in the previous lesson.

The complete phasor diagram is shown in Fig. 14.4b, with the voltage drops across the two components and input (supply) voltage (OA), and also current (OB). The voltage phasor is taken as reference. It may be observed that

$$V_{OC}(= I R) + V_{CA}[= I(j X_L)] = V_{OA}(= I Z),$$

using the Kirchoff's second law relating to the voltage in a closed loop. The phasor diagram can also be drawn with the current phasor as reference, as will be shown in the next lesson.

Power consumed and Power factor

From the waveform of instantaneous power ($W = v \cdot i$) also shown in Fig. 14.4c for the above circuit, the average power is,

$$W = \frac{1}{\pi}\int_0^\pi v \cdot i\, d\theta = \frac{1}{\pi}\int_0^\pi \sqrt{2}\,V \sin\theta \sqrt{2}\,I \sin(\theta - \phi)\, d\theta = \frac{1}{\pi}\int_0^\pi V I\left[\cos\phi - \cos(2\theta - \phi)\right] d\theta$$

$$= \frac{1}{\pi}\left[V I \cos\phi\, \theta\Big|_0^\pi - \frac{V I}{2} \sin(2\theta - \phi)\Big|_0^\pi \right]$$

$$= \frac{1}{\pi}\left[V I \cos\phi\, (\pi - 0) - \frac{V I}{2}\left[\sin(2\pi - \phi) + \sin\phi\right]\right] = V I \cos\phi$$

Note that power is only consumed in resistance, R only, but not in the inductance, L. So, $W = I^2 R$.

$$\text{Power factor} = \frac{average\ power}{apparent\ power} = \frac{V I \cos\phi}{V I} = \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

The power factor in this circuit is less than 1 (one), as $0° \le \phi \le 90°$, ϕ being positive as given above.

For the resistive (R) circuit, the power factor is 1 (one), as $\phi = 0°$, and the average power is $V I$.

For the circuits with only inductance, L or capacitance, C as described earlier, the power factor is 0 (zero), as $\phi = \pm 90°$. For inductance, the phase angle, or the angle of the impedance, $\phi = +90°$ (lagging), and for capacitance, $\phi = -90°$ (leading). It may be noted that in both cases, the average power is zero (0), which means that no power is consumed in the elements, L and C.

The complex power, Volt-Amperes (VA) and reactive power will be discussed after the next section.

2. Capacitive circuit (R and C in series)

This part is discussed in brief. The voltage balance equation for the R-C series circuit (Fig. 14.6a) is,

$$v = R i + \frac{1}{C}\int i\, dt = \sqrt{2}\,V \sin\omega t$$

The current is

$$i = \sqrt{2}\,I \sin(\omega t + \phi)$$

The reasons for the above choice of the current, i, and the steps needed for the derivation of the above expression, have been described in detail, in the case of the earlier example of inductive (R-L) circuit. The same set of steps has to be followed to derive the current, i in this case.

Alternatively, the steps required to find the rms value of the current I, using complex form of impedance, are given here.

The impedance of the capacitive (R-C) circuit is,

$$Z\angle -\phi = R - j X_C = R - j\frac{1}{\omega C}$$

where,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2} \quad \text{and}$$

$$\phi = \tan^{-1}\left(-\dfrac{X_C}{R}\right) = \tan^{-1}\left(-\dfrac{1}{\omega C R}\right) = -\tan^{-1}\left(\dfrac{1}{\omega C R}\right)$$

$$\bar{I}\angle\phi = \dfrac{V\angle 0°}{Z\angle -\phi} = \dfrac{V + j0}{R - jX_C} = \dfrac{V + j0}{R - j(1/\omega C)}$$

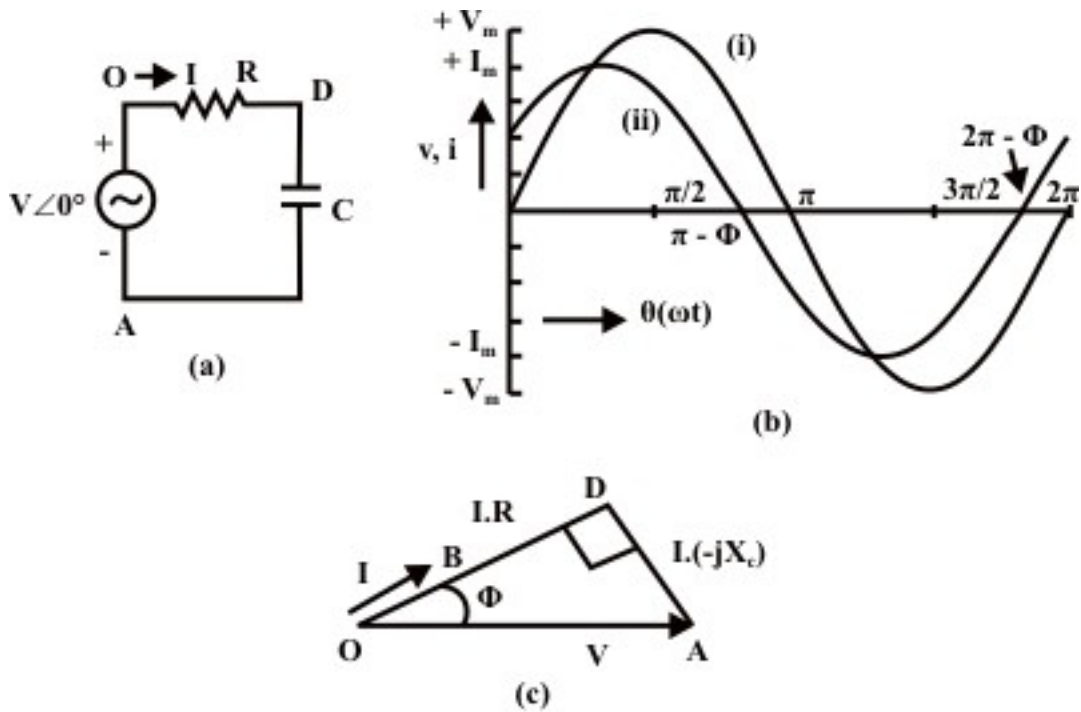$$I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + X_C^2}} = \dfrac{V}{\sqrt{R^2 + (1/\omega C)^2}}$$

Fig. 14.6: Circuit with Resistance (R) and Capacitance (C) in series.
(a) Circuit diagram
(b) Waveforms: (i) Voltage (ii) Current
(c) Phasor diagram

Note that the current leads the voltage by the angle ϕ, value as given above. In this case, the voltage phasor has been taken as reference phase, with the current phasor leading the voltage phasor by the angle, ϕ. But normally, in the case of the series circuit, the current phasor is taken as reference phase, with the voltage phasor lagging the current phasor by ϕ. This can be observed both from phasor diagram (Fig. 14.6b), and waveforms (Fig. 14.6c). The capacitive reactance X_C is negative. In the phasor diagram, as one move from voltage phasor to current phasor, one has to go in the <u>anticlockwise</u> direction, which means that phase angle, ϕ is taken as negative. This is in contrast to the case as described earlier. The complete phasor diagram is shown in Fig. 14.6b, with the voltage drops across the two components and input (supply) voltage, and also current. The voltage phasor is taken as reference.

The power factor in this circuit is less than 1 (one), with ϕ being same as given above. The expression for the average power is $P = V\,I\cos\phi$, which can be obtained by the method shown above. The power is only consumed in the resistance, R, but not in the capacitance, C. One example is included after the next section.

Complex Power, Volt-Amperes (VA) and Reactive Power

The complex power is the product of the voltage and complex conjugate of the current, both in phasor form. For the inductive circuit, described earlier, the voltage ($V \angle 0°$) is taken as reference and the current ($I \angle -\phi = I\cos\phi - j\,I\sin\phi$) is lagging the voltage by an angle, ϕ. The complex power is

$$\bar{S} = \bar{V} \cdot I^* = V\angle 0° \cdot I \angle \phi = (V\,I)\angle\phi = V\,I\cos\ \phi + j\,V\,I\sin\ \phi = P + j\,Q$$

The Volt-Amperes (S), a scalar quantity, is the product of the magnitudes the voltage and the current. So, $S = V \cdot I = \sqrt{P^2 + Q^2}$. It is expressed in VA.

The active power (W) is

$$P = \text{Re}\ (\bar{S}) = \text{Re}\ (\bar{V} \cdot I^*) = V\,I\cos\ \phi, \text{ as derived earlier.}$$

The reactive power (VAr) is given by $Q = \text{Im}\ (\bar{S}) = \text{Im}\ (\bar{V} \cdot I^*) = V\,I\sin\ \phi$.

As the phase angle, ϕ is taken as positive in inductive circuits, the reactive power is positive. The real part, ($I\cos\phi$) is in phase with the voltage V, whereas the imaginary part, $I\sin\phi$ is in quadrature ($-90°$) with the voltage V. But in capacitive circuits, the current ($I \angle\phi$) leads the voltage by an angle ϕ, which is taken as negative. So, it can be stated that the reactive power is negative here, which can easily be derived

Example 14.1

A voltage of 120 V at 50 Hz is applied to a resistance, R in series with a capacitance, C (Fig. 14.7a). The current drawn is 2 A, and the power loss in the resistance is 100 W. Calculate the resistance and the capacitance.

Solution

$V = 120$ V $\qquad I = 2$ A $\qquad P = 100$ W $\quad f = 50$ Hz
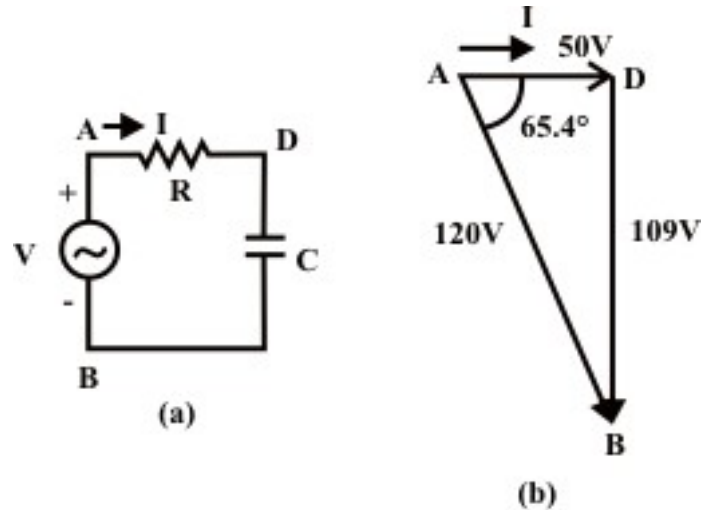
$R = P/I^2 = 100/2^2 = 25\ \Omega$

$Z = \sqrt{R^2 + X_C^2} = V/I = 120/2 = 60\ \Omega$

$X_c = 1/(2\pi f\ C) = \sqrt{Z^2 - R^2} = \sqrt{(60)^2 - (25)^2} = 54.54\ \Omega$

$C = \dfrac{1}{2\pi f\ X_C} = \dfrac{1}{2\pi \cdot 50.0 \times 54.54} = 58.36 \cdot 10^{-6} = 58.36\ \mu F$

The power factor is, $\cos\phi = R/Z = 25/60 = 0.417\ (lead)$

The phase angle is $\phi = \cos^{-1}(0.417) = 65.38°$

Fig. 14.7: (a) Circuit diagram
(b) Phasor diagram

The phasor diagram, with the current as reference, is shown in Fig. 14.7b. The examples, with lossy inductance coil (r in series with L), will be described in the next lesson. The series circuit with all elements, R. L & C, along with parallel circuits, will be taken up in the next lesson.

Problems

14.1 Calculate the power factor in the following cases for the circuit with the elements, as given, fed from a single phase ac supply.

(i) With resistance, R only, but no L and C

(a) 1.0 (Φ=0°)   (b) 0.0 lagging (Φ=+90°)

(c) 0.0 leading (Φ=-90°)   (d) None of the above

(ii) with only pure/lossless inductance, L, but no R and C

(a) 1.0 (Φ=0°)   (b) 0.0 lagging (Φ=+90°)

(c) 0.0 leading (Φ=-90°)   (d) None of the above

(iii) with only pure capacitance, C, but no R and L.

(a) 1.0 (Φ=0°)   (b) 0.0 lagging (Φ=+90°)

(c) 0.0 leading (Φ=-90°)   (d) None of the above

14.2 Calculate the current and power factor (lagging / leading) in the following cases for the circuits having impedances as given, fed from an ac supply of 200 V. Also draw the phasor diagram in all cases.

(i) $Z = (15+j20)$ Ω

(ii) $Z = (14-j14)$ Ω

(iii) $Z = R + j(X_L - X_C)$, where $R = 10$ Ω, $X_L = 20$ Ω, and $X_C = 10$ Ω.

14.3 A 200 V, 50 Hz supply is connected to a resistance (R) of 20 Ω in series with an iron cored choke coil (r in series with L). The readings of the voltmeters across the resistance and across the coil are 120 V and 150 V respectively. Find the loss in the coil. Also find the total power factor. Draw the phasor diagram.

14.4 A circuit, with a resistance, R and a lossless inductance in series, is connected across an ac supply (V) of known frequency (f). A capacitance, C is now connected in series with R-L, with V and f being constant. Justify the following statement with reasons.

The current in the circuit normally increases with the introduction of C.

Under what condition, the current may also decrease. Explain the condition with reasons.

List of Figures

Module
4

Single-phase AC circuits

Lesson
15

Solution of Current in AC Series and Parallel Circuits

In the last lesson, two points were described:

1. How to solve for the impedance, and current in an ac circuit, consisting of single element, R / L / C?

2. How to solve for the impedance, and current in an ac circuit, consisting of two elements, R and L / C, in series, and then draw complete phasor diagram?

In this lesson, the solution of currents in simple circuits, consisting of resistance R, inductance L and/or capacitance C connected in series, fed from single phase ac supply, is presented. Then, the circuit with all above components in parallel is taken up. The process of drawing complete phasor diagram with current(s) and voltage drops in the different components is described. The computation of total power and also power consumed in the different components, along with power factor, is explained. One example of series circuit are presented in detail, while the example of parallel circuit will be taken up in the next lesson.

Keywords: Series and parallel circuits, impedance, admittance, power, power factor.

After going through this lesson, the students will be able to answer the following questions;

1. How to compute the total reactance and impedance / admittance, of the series and parallel circuits, fed from single phase ac supply?

2. How to compute the different currents and also voltage drops in the components, both in magnitude and phase, of the circuit?

3. How to draw the complete phasor diagram, showing the currents and voltage drops?

4. How to compute the total power and also power consumed in the different components, along with power factor?

Solution of Current in R-L-C Series Circuit
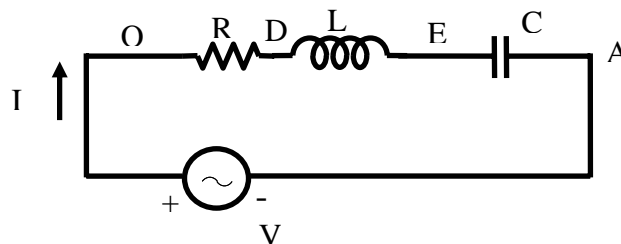
Series (R-L-C) circuit

Fig. 15.1 (a) Circuit diagram

The voltage balance equation for the circuit with R, L and C in series (Fig. 15.1a), is

$$v = R\,i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = \sqrt{2}\,V\sin\omega t$$

The current, i is of the form,

$i = \sqrt{2}\, I \sin\left(\omega t \pm \phi\right)$

As described in the previous lesson (#14) on series (R-L) circuit, the current in steady state is sinusoidal in nature. The procedure given here, in brief, is followed to determine the form of current. If the expression for $i = \sqrt{2}\, I \sin\left(\omega t - \phi\right)$ is substituted in the voltage equation, the equation shown here is obtained, with the sides (LHS & RHS) interchanged.

$R \cdot \sqrt{2}\, I \sin\left(\omega t - \phi\right) + \omega L \cdot \sqrt{2}\, I \cos\left(\omega t - \phi\right) - (1/\omega C) \cdot \sqrt{2}\, I \cos\left(\omega t - \phi\right)$

$= \sqrt{2}\, V \sin \omega t$

or $R \cdot \sqrt{2}\, I \sin\left(\omega t - \phi\right) + [\omega L - (1/\omega C)] \cdot \sqrt{2}\, I \cos\left(\omega t - \phi\right) = \sqrt{2}\, V \sin \omega t$

The steps to be followed to find the magnitude and phase angle of the current I, are same as described there (#14).

So, the phase angle is $\phi = \tan^{-1}[\omega L - (1/\omega C)]/R$

and the magnitude of the current is $I = V/Z$

where the impedance of the series circuit is $Z = \sqrt{R^2 + [\omega L - (1/\omega C)]^2}$

Alternatively, the steps to find the rms value of the current I, using complex form of impedance, are given here.

The impedance of the circuit is

$$Z\angle \pm \phi = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - (1/\omega C)\right)^2} \text{ , and}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - (1/\omega C)}{R}\right)$$

$$\bar{I}\angle \mp \phi = \frac{\bar{V}\angle 0^\circ}{Z\angle \pm \phi} = \frac{V + j0}{R + j(X_L - X_C)} = \frac{V + j0}{R + j\left(\omega L - (1/\omega C)\right)}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

Two cases are: (a) Inductive $\left(\omega L > \dfrac{1}{\omega C}\right)$, and (b) Capacitive $\left(\omega L < \dfrac{1}{\omega C}\right)$.
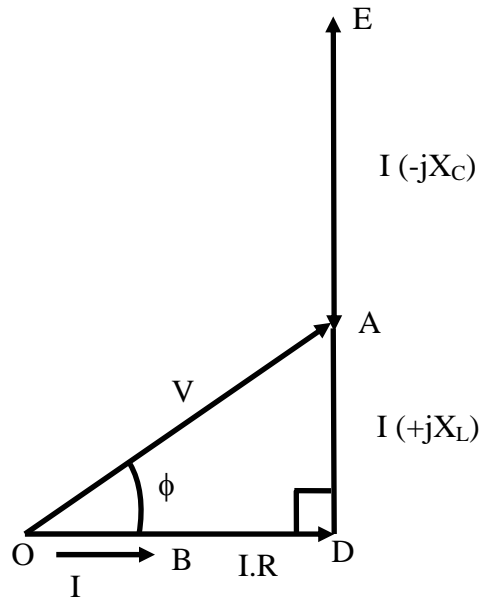
(a) Inductive

In this case, the circuit is inductive, as total reactance $\left(\omega L - (1/\omega C)\right)$ is positive, under the condition $\left(\omega L > (1/\omega C)\right)$. The current lags the voltage by ϕ (taken as positive), with the voltage phasor taken as reference. The power factor (lagging) is less than 1 (one), as $0^\circ \le \phi \le 90^\circ$. The complete phasor diagram, with the voltage drops across the

components and input voltage (OA), and also current (OB), is shown in Fig. 15.1b. The voltage phasor is taken as reference, in all cases. It may be observed that

$$V_{OC}(= i R) + V_{CD}[= i(j X_L)] + V_{DA}[= -i(j X_C)] = V_{OA}(= i Z)$$

using the Kirchoff's second law relating to the voltage in a closed loop. The phasor diagram can also be drawn with the current phasor as reference, as will be shown in the example given here. The expression for the average power is $V I \cos\phi = I^2 R$. The power is only consumed in the resistance, R, but not in inductance/capacitance (L/C), in all three cases.

Inductive ($X_L > X_C$)
Fig 15.1 (b) Phasor diagram

In this case, the circuit is inductive, as total reactance $\left(\omega L - (1/\omega C)\right)$ is positive, under the condition $\left(\omega L > (1/\omega C)\right)$. The current lags the voltage by ϕ (positive). The power factor (lagging) is less than 1 (one), as $0° \le \phi \le 90°$. The complete phasor diagram, with the voltage drops across the components and input voltage (OA), and also current (OB), is shown in Fig. 15.1b. The voltage phasor is taken as reference, in all cases. It may be observed that

$$V_{OC}(= i R) + V_{CD}[= i(j X_L)] + V_{DA}[= -i(j X_C)] = V_{OA}(= i Z)$$

using the Kirchoff's second law relating to the voltage in a closed loop. The phasor diagram can also be drawn with the current phasor as reference, as will be shown in the example given here. The expression for the average power is $V I \cos\phi = I^2 R$. The power is only consumed in the resistance, R, but not in inductance/capacitance (L/C), in all three cases.

(b) Capacitive

Capacitive ($X_L < X_C$)
Fig 15.1 (c) Phasor diagram

The circuit is now capacitive, as total reactance $\left(\omega L - (1/\omega C)\right)$ is negative, under the condition $\left(\omega L < (1/\omega C)\right)$. The current leads the voltage by ϕ, which is negative as per convention described in the previous lesson. The voltage phasor is taken as reference here. The complete phasor diagram, with the voltage drops across the components and input voltage, and also current, is shown in Fig. 15.1c. The power factor (leading) is less than 1 (one), as $0° \leq \phi \leq 90°$, ϕ being negative. The expression for the average power remains same as above.

The third case is resistive, as total reactance $(\omega L - 1/\omega C)$ is zero (0), under the condition $(\omega L = 1/\omega C)$. The impedance is $Z \angle 0° = R + j0$. The current is now at unity power factor ($\phi = 0°$), i.e. the current and the voltage are in phase. The complete phasor diagram, with the voltage drops across the components and input (supply) voltage, and also current, is shown in Fig. 15.1d. This condition can be termed as 'resonance' in the series circuit, which is described in detail in lesson #17. The magnitude of the impedance in the circuit is minimum under this condition, with the magnitude of the current being maximum. One more point to be noted here is that the voltage drops in the inductance, L and also in the capacitance, C, is much larger in magnitude than the supply voltage, which is same as the voltage drop in the resistance, R. The phasor diagram has been drawn approximately to scale.
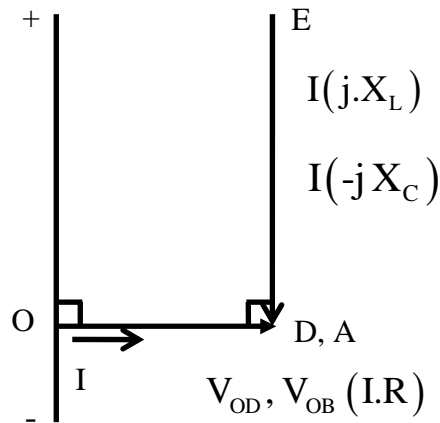
Resistive (X$_L$ = X$_C$)

Fig. 15.1 (d) Phasor diagram

It may be observed here that two cases of series (R-L & R-C) circuits, as discussed in the previous lesson, are obtained in the following way. The first one (inductive) is that of (a), with C very large, i.e. $1/\omega C \approx 0$, which means that C is not there. The second one (capacitive) is that of (b), with L not being there (L or $\omega L = 0$).

Example 15.1

A resistance, R is connected in series with an iron-cored choke coil (r in series with L). The circuit (Fig. 15.2a) draws a current of 5 A at 240 V, 50 Hz. The voltages across the resistance and the coil are 120 V and 200 V respectively. Calculate,
    (a) the resistance, reactance and impedance of the coil,
    (b) the power absorbed by the coil, and
    (c) the power factor (pf) of the input current.
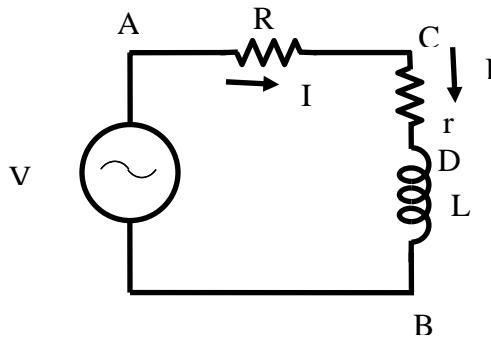
Fig. 15.2 (a) Circuit diagram

Solution

$I\,(OB) = 5$ A $V_S\,(OA) = 240$ V $f = 50$ Hz $\omega = 2\pi f$

The voltage drop across the resistance $V_1\,(OC) = I \cdot R = 120$ V

The resistance, $R = V_1 / I = 120 / 5 = 24 \ \Omega$

The voltage drop across the coil $V_2 \ (CA) = I \cdot Z_L = 200 \ V$

The impedance of the coil, $Z_L = \sqrt{r^2 + X_L^2} = V_2 / I = 200 / 5 = 40 \ \Omega$

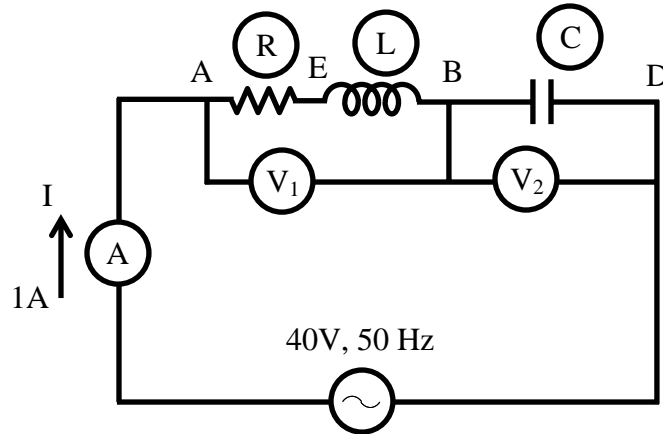From the phasor diagram (Fig. 15.2b),

Fig. 15.2(b): Phasor Diagram

$$\cos \phi = \cos \angle AOC = \frac{OA^2 + OC^2 - CA^2}{2 \cdot OA \cdot OC} = \frac{(120)^2 + (240)^2 - (200)^2}{2 \times 120 \times 240} = \frac{32,000}{57,600}$$

$= 0.556$

The power factor (pf) of the input current $= \cos \phi = 0.556 \ (lag)$

The phase angle of the total impedance, $\phi = \cos^{-1} (0.556) = 56.25°$

Input voltage, $V_S \ (OA) = I \cdot Z = 240 \ V$

The total impedance of the circuit, $Z = \sqrt{(R + r)^2 + X_L^2} = V_S / I = 240 / 5 = 48 \ \Omega$

$Z \angle \phi = (R + r) + j X_L = 48 \ \angle 56.25° = (26.67 + j 39.91) \ \Omega$

The total resistance of the circuit, $R + r = 24 + r = 26.67 \ \Omega$

The resistance of the coil, $r = 26.67 - 24.0 = 2.67 \ \Omega$

The reactance of the coil, $X_L = \omega L = 2 \ \pi \ f \ L = 39.9 \ \Omega$

The inductance of the coil, $L = \dfrac{X_L}{2 \pi f} = \dfrac{39.9}{2 \pi \times 50} = 0.127 \ H = 127 \cdot 10^{-3} = 127 \ mH$

The phase angle of the coil,

$\phi_L = \cos^{-1} (r / Z_L) = \cos^{-1} (2.67 / 40.0) = \cos^{-1} (0.067) = 86.17°$

$Z_L \angle \phi_L = r + j X_L = (2.67 + j \ 39.9) = 40 \ \angle 86.17°7) \ \Omega$

The power factor (pf) of the coil, $\cos \phi \ = 0.067 \ (lag)$

The copper loss in the coil $= I^2 r = 5^2 \times 2.67 = 66.75 \ W$

Example 15.2

An inductive coil, having resistance of 8 Ω and inductance of 80 mH, is connected in series with a capacitance of 100 μF across 150 V, 50 Hz supply (Fig. 15.3a). Calculate, (a) the current, (b) the power factor, and (c) the voltages drops in the coil and capacitance respectively.
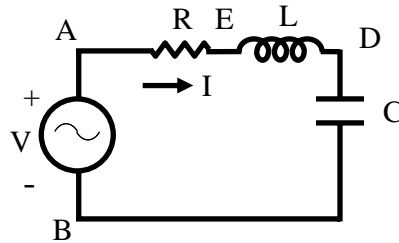

Fig. 15.3 (a) Circuit diagram

Solution

$$f = 50 \text{ Hz} \qquad\qquad \omega = 2\pi f = 2\pi \times 50 = 314.16 \; rad/s$$

$$\text{L} = 80 \; mH = 80 \cdot 10^{-3} = 0.08 \, H \qquad\qquad X_l = \omega L = 314.16 \times 0.08 = 25.13 \; \Omega$$

$$C = 100 \; \mu F = 100 \cdot 10^{-6} \, F \qquad X_C = \frac{1}{\omega C} = \frac{1}{314.16 \times 100 \cdot 10^{-6}} = 31.83 \; \Omega$$

$$\text{R} = 8 \; \Omega \qquad\qquad V_S \, (OA) = 150 \; V$$

The impedance of the coil, $Z_L \angle\phi_L = R + j X_L = (8.0 + j\,25.13) = 26.375\angle 72.34° \, \Omega$

The total impedance of the circuit,

$$Z\angle-\phi = R + j(X_L - X_{C4}) = 8.0 + j(25.13 - 31.83) = (8.0 - j\,6.7)$$

$$= 10.435\angle-39.95° \; \Omega$$

The current drawn from the supply,

$$I\angle\phi = \frac{V\angle 0°}{Z\angle-\phi} = \left(\frac{150}{10.435}\right)\angle 39.95° = 14.375\angle 39.95° \; A = (11.02 + j\,9.26) \; A$$

The current is, $I = 14.375 \; A$

The power factor (pf) $= \cos\phi = \cos 39.95° = 0.767 \; (lead)$
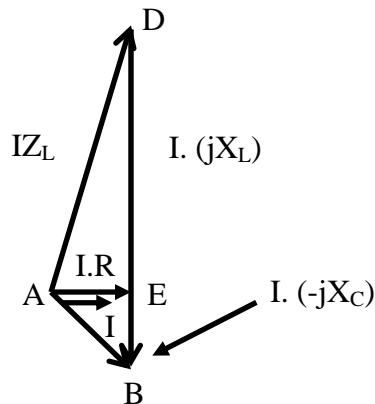

Fig. 15.3 (b) Phasor diagram

Please note that the current phasor is taken as reference in the phasor diagram (Fig. 15.3b) and also here. The voltage drop in the coil is,

$$V_1 \angle \theta_1 = I \angle 0° \cdot Z_L \angle \phi_L = (14.375 \times 26.375) \angle 72.34° = 379.14 \angle 72.34° \ V$$
$$= (115.1 + j\,361.24) \ V$$

The voltage drop in the capacitance in,

$$V_2 \angle \theta_2 = I \angle 0° \cdot Z_C \angle -\phi_C = (14.475 \times 31.83) \angle -90.0° = 457.58 \angle -90.0° \ V$$
$$= -j\,457.58 \ V$$

Solution of Current in Parallel Circuit

Parallel circuit

The circuit with all three elements, R, L & C connected in parallel (Fig. 15.4a), is fed to the ac supply. The current from the supply can be computed by various methods, of which two are described here.
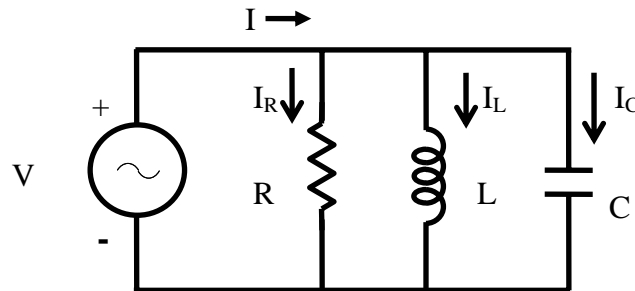

Fig. 15.4 (a) Circuit diagram.

First method

The current in three branches are first computed and the total current drawn from the supply is the phasor sum of all three branch currents, by using Kirchoff's first law related to the currents at the node. The voltage phasor (V) is taken as reference.

All currents, i.e. three branch currents and total current, in steady state, are sinusoidal in nature, as the input (supply voltage is sinusoidal of the form,

$$v = \sqrt{2} \ V \sin \omega t$$

Three branch currents are obtained by the procedure given in brief.

$$v = R \cdot i_R \ , \text{ or } i_R = v / R = \sqrt{2} \ (V / R) \sin \omega t = \sqrt{2} \ I_R \sin \omega t \ ,$$

where, $|I_R| = |(V / R)|$

Similarly, $v = L \dfrac{d\,i_L}{dt}$

So, i_L is,

$$i_L = (1/L) \int v \ dt = (1/L) \int \sqrt{2} \ V (\sin \omega t) \ dt = -\sqrt{2} \,[V /(\omega L)] \cos \omega t = -\sqrt{2} \ I_L \cos \omega t$$
$$= \sqrt{2} \ I_L \sin (\omega t - 90°)$$

where, $\left|I_L\right| = \left|(V/X_L)\right|$ with $X_L = \omega L$

$v = (1/C)\int i_C\,dt$, from which i_C is obtained as,

$i_C = C\dfrac{dv}{dt} = C\dfrac{d}{dt}(\sqrt{2}\,V\sin\omega t) = \sqrt{2}\,(V\cdot\omega C)\cos\omega t = \sqrt{2}\,I_C\cos\omega t$

$= \sqrt{2}\,I_C\sin(\omega t + 90°)$

where, $\left|I_c\right| = \left|(V/X_C)\right|$ with $X_C = (1/\omega C)$

Total (supply) current, i is

$i = i_R + i_L + i_C = \sqrt{2}\,I_R\sin\omega t - \sqrt{2}\,I_L\cos\omega t + \sqrt{2}\,I_C\cos\omega t$

$= \sqrt{2}\,I_R\sin\omega t - \sqrt{2}\,(I_L - I_C)\cos\omega t = \sqrt{2}\,I\sin(\omega t \mp \phi)$

The two equations given here are obtained by expanding the trigonometric form appearing in the last term on RHS, into components of $\cos\omega t$ and $\sin\omega t$, and then equating the components of $\cos\omega t$ and $\sin\omega t$ from the last term and last but one (previous).

$I\cos\phi = I_R$ and $I\sin\phi = (I_L - I_C)$

From these equations, the magnitude and phase angle of the total (supply) current are,

$|I| = \sqrt{(I_R)^2 + (I_L - I_C)^2} = |V|\cdot\sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{X_L} - \dfrac{1}{X_C}\right)^2}$

$= |V|\cdot\sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{\omega L} - \omega C\right)^2} = |V|\cdot|Y|$

$\phi = \tan^{-1}\left(\dfrac{I_L - I_C}{I_R}\right) = \tan^{-1}\left(\dfrac{(1/X_L) - (1/X_C)}{(1/R)}\right) = \tan^{-1}\left[R\cdot\left(\dfrac{1}{X_l} - \dfrac{1}{X_C}\right)\right]$

$= \tan^{-1}\left[R\cdot\left(\dfrac{1}{\omega L} - \omega C\right)\right]$

where, the magnitude of the term (admittance of the circuit) is,

$|Y| = \sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{X_L} - \dfrac{1}{X_C}\right)^2} = \sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{\omega L} - \omega C\right)^2}$

Please note that the admittance, which is reciprocal of impedance, is a complex quantity. The angle of admittance or impedance, is same as the phase angle, ϕ of the current I, with the input (supply) voltage taken as reference phasor, as given earlier.

Alternatively, the steps required to find the rms values of three branch currents and the total (suuply) current, using complex form of impedance, are given here.

Three branch currents are

$I_R\angle 0° = I_R = \dfrac{V}{R}\,;\quad I_L\angle -90° = -j\,I_L = \dfrac{V}{j\,X_L} = \dfrac{V}{j\omega L} = -j\,\dfrac{V}{\omega L}$

$I_C\angle +90° = j\,I_C = \dfrac{V}{-j\,X_C} = \dfrac{V}{-j\,(1/\omega C)} = j\omega C V$

Of the three branches, the first one consists of resistance only, the current, I_R is in phase with the voltage (V). In the second branch, the current, I_L lags the voltage by $90°$, as there is inductance only, while in the third one having capacitance only, the current, I_C leads the voltage $90°$. All these cases have been presented in the previous lesson.

The total current is

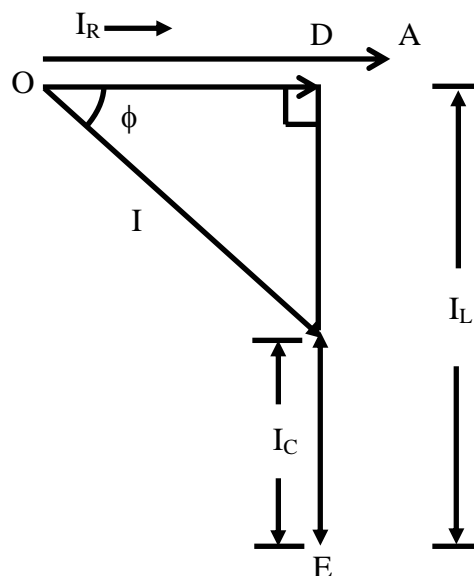$$I \angle \pm \phi = I_R + j(I_c - I_L) = V \left[\frac{1}{R} + j \left(\omega C - \frac{1}{\omega L} \right) \right]$$

where,

$$I = \sqrt{I_R^2 + (I_C - I_L)^2} = V \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]} \text{ , and}$$

$$\phi = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right) = \tan^{-1} \left[R \left(\omega C - \frac{1}{\omega L} \right) \right]$$

The two cases are as described earlier in series circuit.

(a) Inductive

Inductive ($I_L > I_C$)
Fig. 15.4 (b) Phasor diagram

In this case, the circuit being inductive, the current lags the voltage by ϕ (positive), as $I_L > I_C$, i.e. $1/\omega L > \omega C$, or $\omega L < 1/\omega C$ .This condition is in contrast to that derived in the case of series circuit earlier. The power factor is less than 1 (one). The complete phasor diagram, with the three branch currents along with total current, and also the voltage, is shown in Fig. 15.4b. The voltage phasor is taken as reference in all cases. It may be observed there that

$$I_R(OD) + I_L(DC) + I_C(CB) = I(OB)$$

The Kirchoff's first law related to the currents at the node is applied, as stated above. The expression for the average power is $V I \cos\phi = I_R^2 R = V^2 / R$. The power is only consumed in the resistance, R, but not in inductance/capacitance (L/C), in all three cases.

(b) Capacitive

The circuit is capacitive, as $I_C > I_L$, i.e. $\omega C > 1/\omega L$, or $\omega L > 1/\omega C$. The current leads the voltage by ϕ (ϕ being negative), with the power factor less than 1 (one). The complete phasor diagram, with the three branch currents along with total current, and also the voltage, is shown in Fig. 15.4c.

The third case is resistive, as $|I_L| = |I_C|$, i.e. $1/\omega L = \omega C$ or $\omega L = 1/\omega C$. This is the same condition, as obtained in the case of series circuit. It may be noted that two currents, I_L and I_C, are equal in magnitude as shown, but opposite in sign (phase difference being $180°$), and the sum of these currents $(I_L + I_C)$ is zero (0). The total current is in phase with the voltage ($\phi = 0°$), with $|I| = |I_R|$, the power factor being unity. The complete phasor diagram, with the three branch currents along with total current, and also the voltage, is shown in Fig. 15.4d. This condition can be termed as 'resonance' in the parallel circuit, which is described in detail in lesson #17. The magnitude of the impedance in the circuit is maximum (i.e., the magnitude of the admittance is minimum) under this condition, with the magnitude of the total (supply) current being minimum.
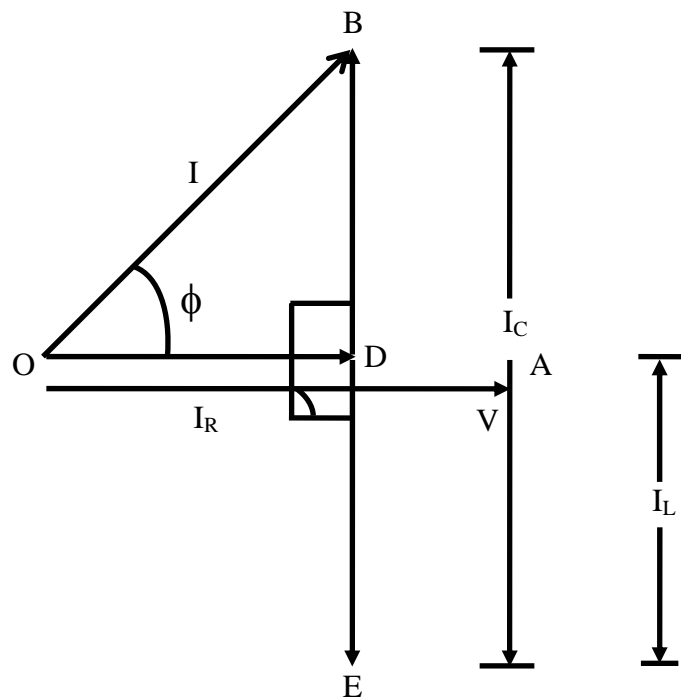
Capacitive ($I_L < I_C$)
Fig. 15.4 (c) Phasor diagram

The circuit with two elements, say R & L, can be solved, or derived with C being large ($I_C = 0$ or $1/\omega C = 0$).
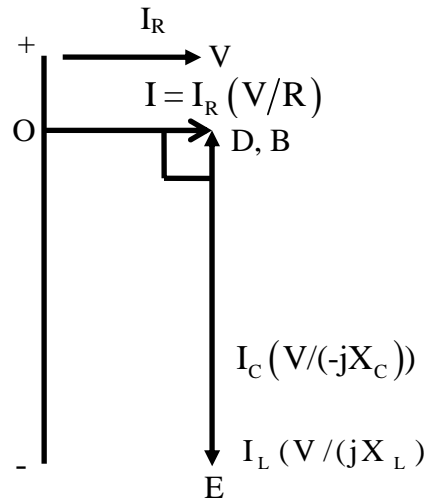
Resistive ($\mathbf{I_L = I_C}$)

Fig. 15.4 (d) Phasor Diagram

Second method

Before going into the details of this method, the term, <u>Admittance</u> must be explained. In the case of two resistance connected in series, the equivalent resistance is the sum of two resistances, the resistance being scalar (positive). If two impedances are connected in series, the equivalent impedance is the sum of two impedances, all impedances being complex. Please note that the two terms, real and imaginary, of two impedances and also the equivalent one, may be positive or negative. This was explained in lesson no. 12.

If two resistances are connected in parallel, the inverse of the equivalent resistance is the sum of the inverse of the two resistances. If two impedances are connected in parallel, the inverse of the equivalent impedance is the sum of the inverse of the two impedances. The inverse or reciprocal of the impedance is termed 'Admittance', which is complex. Mathematically, this is expressed as

$$Y = \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = Y_1 + Y_2$$

As admittance (Y) is complex, its real and imaginary parts are called conductance (G) and susceptance (B) respectively. So, $Y = G + jB$. If impedance, $Z \angle \phi = R + jX$ with X being positive, then the admittance is

$$Y \angle -\phi = \frac{1}{Z \angle 0°} = \frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R - jX}{R^2 + X^2}$$

$$= \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G - jB$$

where,

$$G = \frac{R}{R^2 + X^2} \; ; \quad B = \frac{X}{R^2 + X^2}$$

Please note the way in which the result of the division of two complex quantities is obtained. Both the numerator and the denominator are multiplied by the complex conjugate of the denominator, so as to make the denominator a real quantity. This has also been explained in lesson no. 12.

The magnitude and phase angle of Z and Y are

$$Z = \sqrt{R^2 + X^2} \; ; \quad \phi = \tan^{-1}(X / R) \quad , \text{and}$$

$$Y = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}} \; ; \quad \phi = \tan^{-1}\left(\frac{B}{G}\right) = \tan^{-1}\left(\frac{X}{R}\right)$$

To obtain the <u>current</u> in the circuit (Fig. 15.4a), the steps are given here.
The admittances of the three branches are

$$Y_1 \angle 0° = \frac{1}{Z_1} = \frac{1}{R} \; ; \quad Y_2 \angle -90° = \frac{1}{Z_2} = \frac{1}{j X_L} = -j \frac{1}{\omega L}$$

$$Y_3 \angle 90° = \frac{1}{Z_3} = \frac{1}{-j X_C} = j \omega C$$

The total admittance, obtained by the phasor sum of the three branch admittances, is

$$Y \angle \pm \phi = Y_1 + Y_2 + Y_3 = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = G + j B$$

where,

$$Y = \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]} \; ; \quad \phi = \tan^{-1}\left[R\left(\omega C - \frac{1}{\omega L}\right)\right] \quad , \text{and}$$

$$G = 1/R \; ; \quad B = \omega C - 1/\omega L$$

The total impedance of the circuit is

$$Z \angle \mp \phi = \frac{1}{Y \angle \pm \phi} = \frac{1}{G + j B} = \frac{G}{G^2 + B^2} - j \frac{B}{G^2 + B^2}$$

The total current in the circuit is obtained as

$$I \angle \pm \phi = \frac{V \angle 0°}{Z \angle \mp \phi} = V \angle 0° \cdot Y \angle \pm \phi = (V Y) \angle \pm \phi$$

where the magnitude of current is $I = V \cdot Y = V / Z$
The current is the same as obtained earlier, with the value of Y substituted in the above equation.

This is best illustrated with an example, which is described in the next lesson.

The solution of the current in the series-parallel circuits will also be discussed there, along with some examples.

Problems

15.1 Calculate the current and power factor (lagging / leading) for the following circuits (Fig. 15.5a-d), fed from an ac supply of 200 V. Also draw the phasor diagram in all cases.
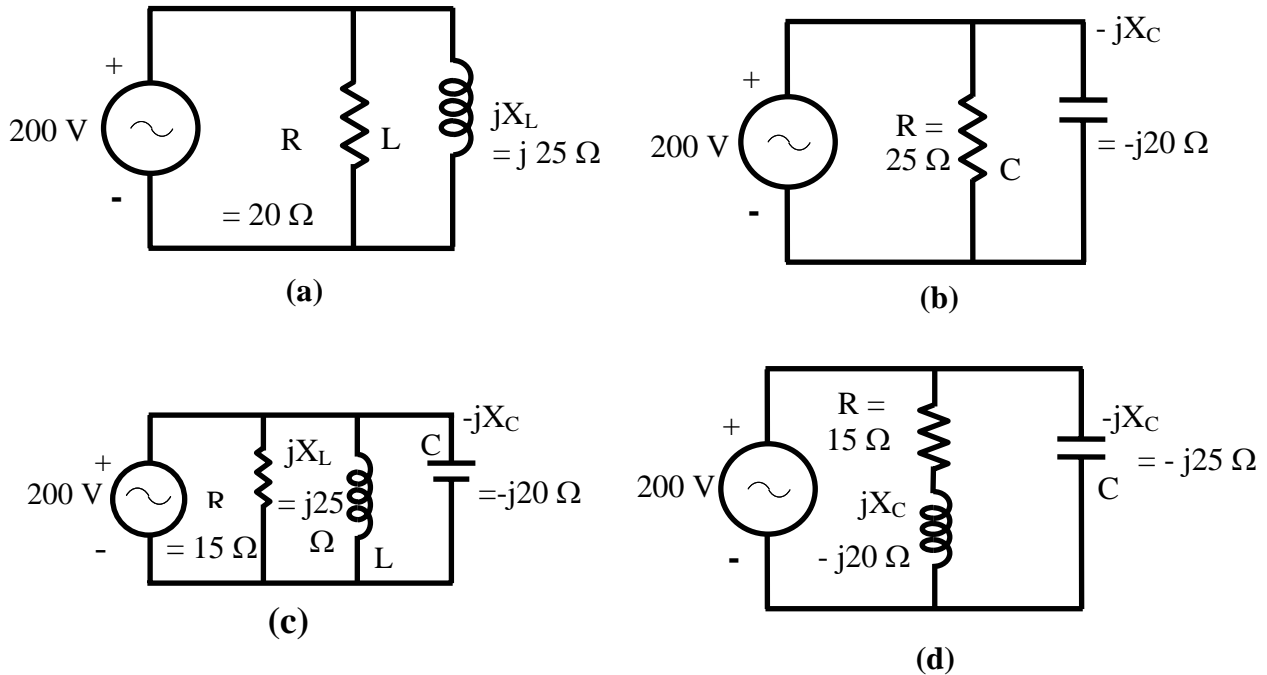

(a)

(b)

(c)

(d)

Fig. 15.5

15.2 A voltage of 200 V is applied to a pure resistor (R), a pure capacitor, C and a lossy inductor coil, all of them connected in parallel. The total current is 2.4 A, while the component currents are 1.5, 2.0 and 1.2 A respectively. Find the total power factor and also the power factor of the coil. Draw the phasor diagram.

15.3 A 200 V. 50Hz supply is connected to a lamp having a rating of 100 V, 200 W, in series with a pure inductance, L, such that the total power consumed is the same, i.e. 200W. Find the value of L.

A capacitance, C is now connected across the supply. Find value of C, to bring the supply power factor to unity (1.0). Draw the phasor diagram in the second case.

1.(a) Find the value of the load resistance (R_L) to be connected in series with a real voltage source ($V_S + R_S$ in series), such that maximum power is transferred from the above source to the load resistance.

   (b) Find the voltage was 8Ω resistance in the circuit shown in Fig. 1(b).

2.(a) Find the Theremin's equivalent circuit (draw the ckt.) between the terminals A + B, of the circuit shown in Fig. 2(a).

(b) A circuit shown in Fig. 2(b) is supplied at 40V, 50Hz. The two voltages V_1 and V_2 (magnitude only) is measured as 60V and 25V respectively. If the current, I is measured as 1A, find the values of R, L and C. Also find the power factor of the circuit (R-L-C). Draw the complete phasor diagram.

3.(a) Find the line current, power factor, and active (real) power drawn from 3-phase, 100V, 50Hz, balanced supply in the circuit shown in Fig. 3(a).

(b) In the circuit shown in Fig. 3(b), the switch, S is put in position 1 at t = 0. Find $i_e(t)$, t > 0, if $v_c(0^-) = 6V$. After the circuit reaches steady state, the switch, S is brought to position 2, at t = T_1. Find $i_c(t)$, t > T_1. Switch the above waveform.

4.(a) Find the average and rms values of the periodic waveform shown in Fig. 4(a).

(b) A coil of 1mH lowing a series resistance of 1Ω is connected in parallel with a capacitor, C and the combination is fed from 100 mV (0.1V), 1 kHz supply (source) having an internal resistance of 10Ω. If the circuit draws power at unity power factor (upf), determine the value of the capacitor, quality factor of the coil, and power drawn by the circuit. Also draw the phasor diagram.

List of Figures

Module
4

Single-phase AC Circuits

Lesson
17

Resonance in Series and Parallel Circuits

In the last lesson, the following points were described:

1. How to compute the total impedance in parallel and series-parallel circuits?

2. How to solve for the current(s) in parallel and series-parallel circuits, fed from single phase ac supply, and then draw complete phasor diagram?

3. How to find the power consumed in the circuits and also the different components, and the power factor (lag/lead)?

In this lesson, the phenomenon of the resonance in series and parallel circuits, fed from single phase variable frequency supply, is presented. Firstly, the conditions necessary for resonance in the above circuits are derived. Then, the terms, such as bandwidth and half power frequency, are described in detail. Some examples of the resonance conditions in series and parallel circuits are presented in detail, along with the respective phasor diagrams.

Keywords: Resonance, bandwidth, half power frequency, series and parallel circuits,

After going through this lesson, the students will be able to answer the following questions;

1. How to derive the conditions for resonance in the series and parallel circuits, fed from a single phase variable frequency supply?

2. How to compute the bandwidth and half power frequency, including power and power factor under resonance condition, of the above circuits?

3. How to draw the complete phasor diagram under the resonance condition of the above circuits, showing the currents and voltage drops in the different components?

Resonance in Series and Parallel Circuits
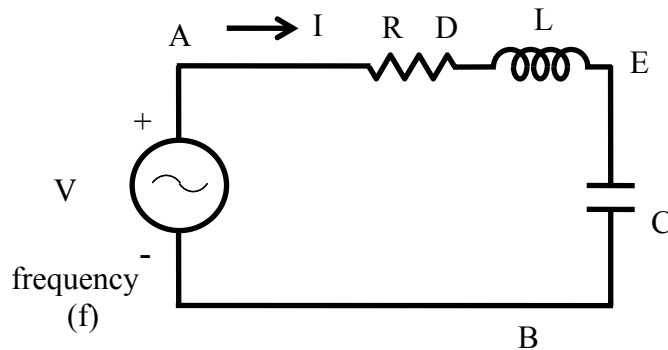
Series circuit

Fig. 17.1 (a) Circuit diagram.

The circuit, with resistance R, inductance L, and a capacitor, C in series (Fig. 17.1a) is connected to a single phase variable frequency (f) supply.

The total impedance of the circuit is

$$Z \angle \phi = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where,

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} \quad ; \quad \phi = \tan^{-1}\frac{(\omega L - 1/\omega C)}{R} \quad ; \quad \omega = 2\pi f$$

The current is

$$I \angle -\phi = \frac{V \angle 0°}{Z \angle \phi} = (V/Z)\angle -\phi$$

where $I = \dfrac{V}{\left[R^2 + \left(\omega L - (1/\omega C)^2\right]^{\frac{1}{2}}}$

The current in the circuit is maximum, if $\omega L = \dfrac{1}{\omega C}$.

The frequency under the above condition is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

This condition under the magnitude of the current is maximum, or the magnitude of the impedance is minimum, is called resonance. The frequency under this condition with the constant values of inductance L, and capacitance C, is called resonant frequency. If the capacitance is variable, and the frequency, f is kept constant, the value of the capacitance needed to produce this condition is

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$$

The magnitude of the impedance under the above condition is $|Z| = R$, with the reactance $X = 0$, as the inductive reactance $X_l = \omega L$ is equal to capacitive reactance $X_C = 1/\omega C$. The phase angle is $\phi = 0°$, and the power factor is unity ($\cos\phi = 1$), which means that the current is in phase with the input (supply) voltage.. So, the magnitude of the current ($|(V/R)|$) in the circuit is only limited by resistance, R. The phasor diagram is shown in Fig. 17.1b.

The magnitude of the voltage drop in the inductance L/capacitance C (both are equal, as the reactance are equal) is $I \cdot \omega_o L = I \cdot (1/\omega_o C)$.

The magnification of the voltage drop as a ratio of the input (supply) voltage is

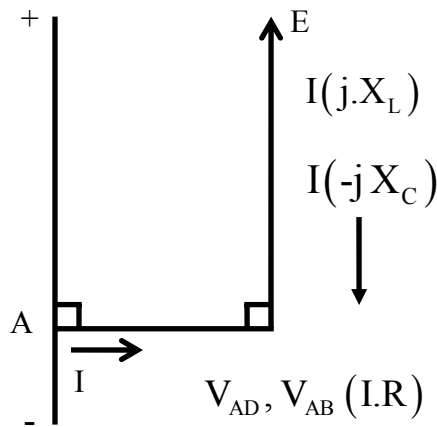$$Q = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Fig. 17.1 (b) Phasor Diagram

It is termed as Quality (Q) factor of the coil.

The impedance of the circuit with the constant values of inductance L, and capacitance C is minimum at resonant frequency (f_o), and increases as the frequency is changed, i.e. increased or decreased, from the above frequency. The current is maximum at $f = f_o$, and decreases as frequency is changed ($f > f_o$, or $f < f_o$), i.e. $f \neq f_o$. The variation of current in the circuit having a known value of capacitance with a variable frequency supply is shown in Fig. 17.2.

(a)

(b)
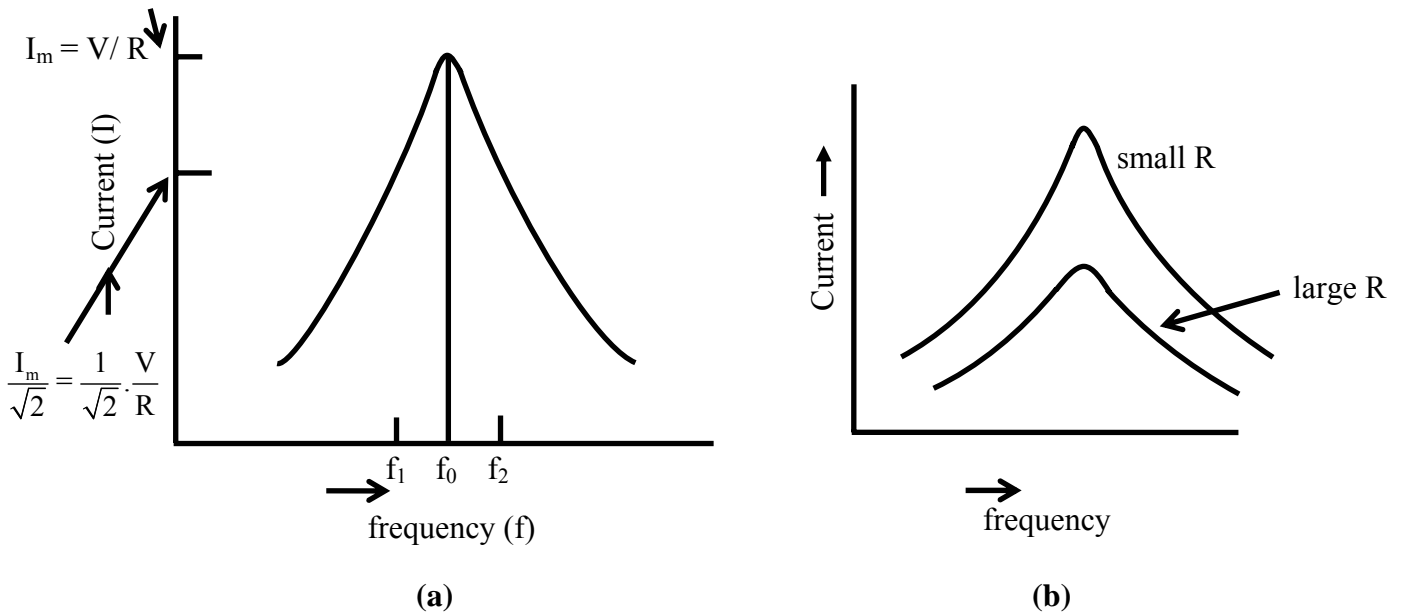
Fig. 17.2 Variation of current under variable frequency supply

The maximum value of the current is (V/R). If the magnitude of the current is reduced to ($1/\sqrt{2}$) of its maximum value, the power consumed in R will be half of that with the maximum current, as power is I^2R. So, these points are termed as half power

points. If the two frequencies are taken as f_1 and f_2, where $f_1 = f_0 - \Delta f / 2$ and $f_2 = f_0 + \Delta f / 2$, the band width being given by $\Delta f = f_2 - f_1$.

The magnitude of the impedance with the two frequencies is

$$Z = \left[R^2 + \left(2\pi(f_0 \pm \Delta f / 2)L - \frac{1}{2\pi(f_0 \pm \Delta f / 2)C} \right)^2 \right]^{\frac{1}{2}}$$

As $(2\pi f_0 L = 1/2\pi f_0 C)$ and the ratio $(\Delta f / 2 f_0)$ is small, the magnitude of the reactance of the circuit at these frequencies is $X = X_{L0}(\Delta f / f_0)$. As the current is $(1/\sqrt{2})$ of its maximum value, the magnitude of the impedance is $(\sqrt{2})$ of its minimum value (R) at resonant frequency.

So, $Z = \sqrt{2} \cdot R = \left[R^2 + \left(X_{L0}(\Delta f / f_0) \right)^2 \right]^{\frac{1}{2}}$

From the above, it can be obtained that $(\Delta f / f_0)X_{L0} = R$

or $\Delta f = f_2 - f_1 = \dfrac{R f_0}{X_{L0}} = \dfrac{R f_0}{2\pi f_0 L} = \dfrac{R}{2\pi L}$

The band width is given by $\Delta f = f_2 - f_1 = R / (2\pi L)$

It can be observed that, to improve the quality factor (Q) of a coil, it must be designed to have its resistance, R as low as possible. This also results in reduction of band width and losses (for same value of current). But if the resistance, R cannot be decreased, then Q will decrease, and also both band width and losses will increase.

Example 17.1

A constant voltage of frequency, 1 MHz is applied to a lossy inductor (r in series with L), in series with a variable capacitor, C (Fig. 17.3). The current drawn is maximum, when C = 400 pF; while current is reduced to $(1/\sqrt{2})$ of the above value, when C = 450 pF. Find the values of r and L. Calculate also the quality factor of the coil, and the bandwidth.
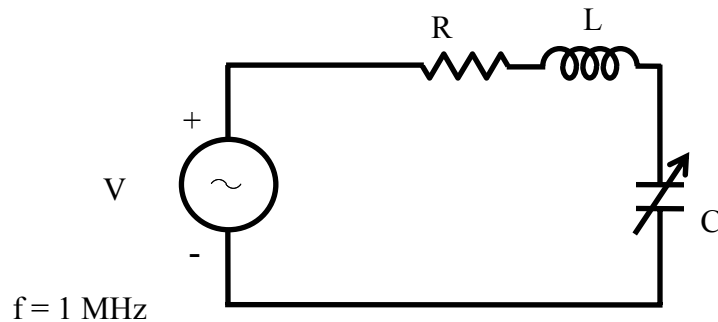

Fig. 17.3 Circuit diagram

Solution

$f = 1 \text{ MHz} = 10^6 \text{ Hz}$ $\qquad \omega = 2\pi f \qquad C = 400\, pF = 400 \cdot 10^{-12}\, F$

$$I_{max} = V / r \quad \text{as} \quad X_L = X_C \qquad X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 10^6 \times 400 \cdot 10^{-12}} = 398 \ \Omega$$

$$X_L = X_C = 2\pi f L = 398 \ \Omega \qquad L = \frac{398.0}{2\pi \cdot 10^6} = 63.34 \ \mu H$$

$$C_1 = 450 \ pF \qquad X_{C1} = = \frac{1}{2\pi \cdot 10^6 \times 450 \cdot 10^{-12}} = 353.7 \ \Omega$$

$$Z\angle\phi = r + j\left(X_L - X_{C1}\right) = r + j\left(398.0 - 353.7\right) = (r + j\,44.3) \ \Omega$$

$$I = \frac{I_{max}}{\sqrt{2}} = \frac{V}{\sqrt{2}\cdot r} = \frac{V}{Z} = \frac{V}{\sqrt{r^2 + (44.3)^2}}$$

From above, $\sqrt{2}\cdot r = \sqrt{r^2 + (44.3)^2} \qquad \text{or} \ 2r^2 = r^2 + (44.3)^2$

or $r = 44.3 \ \Omega$

The quality factor of the coil is $Q = \dfrac{X_L}{r} = \dfrac{398.0}{44.3} = 8.984$

The band with is

$$\Delta f = f_2 - f_1 = \frac{r}{2\pi L} = \frac{44.3}{2\pi \times 63.34 \cdot 10^{-6}} = \frac{44.3}{398 \cdot 10^{-6}} = 0.1113 \cdot 10^6 = 0.1113 \ MHz$$

$$= 111.3 \cdot 10^3 = 111.3 \ kHz$$

Parallel circuit

The circuit, with resistance R, inductance L, and a capacitor, C in parallel (Fig. 17.4a) is connected to a single phase variable frequency (f) supply.

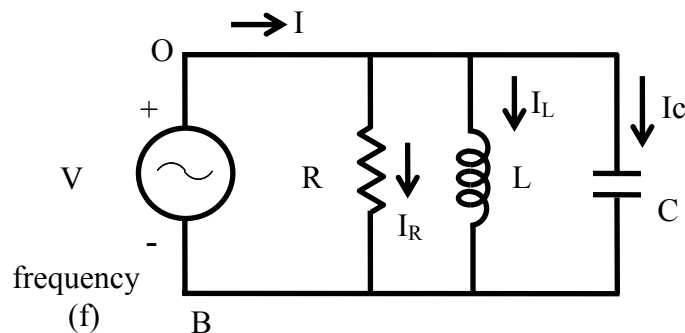The total admittance of the circuit is

Fig. 17.4 (a) Circuit diagram.

$$Y\angle\phi = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

where,

$$Y = \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]} \; ; \quad \phi = \tan^{-1}\left[R\left(\omega C - \frac{1}{\omega L}\right)\right] \; ; \quad \omega = 2\pi f$$

The impedance is $Z \angle -\phi = 1/Y \angle \phi$

The current is

$$I \angle \phi = V \angle 0° \cdot Y \angle \phi = (V \cdot Y) \angle \phi = V \angle 0° / Z \angle -\phi = (V/Z) \angle \phi$$

where, $I = V \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]}$

The current in the circuit is minimum, if $\omega C = \dfrac{1}{\omega L}$

The frequency under the above condition is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$
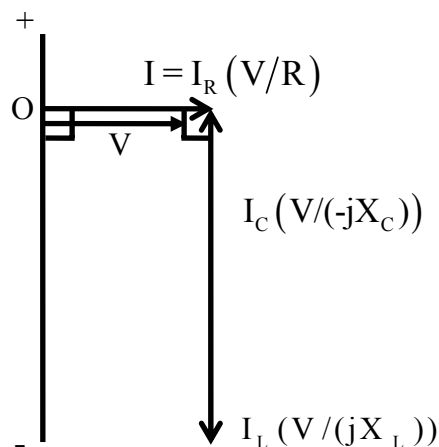
Fig. 17.4 (b) Phasor Diagram

This condition under which the magnitude of the total (supply) current is minimum, or the magnitude of the admittance is minimum (which means that the impedance is maximum), is called resonance. It may be noted that, for parallel circuit, the current or admittance is minimum (the impedance being maximum), while for series circuit, the current is maximum (the impedance being minimum). The frequency under this condition with the constant values of inductance L, and capacitance C, is called resonant frequency. If the capacitance is variable, and the frequency, f is kept constant, the value of the capacitance needed to produce this condition is

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$$

The magnitude of the impedance under the above condition is ($|Z| = R$), while the magnitude of the admittance is ($|Y| = G = (1/R)$). The reactive part of the admittance is

$B = 0$, as the susceptance (inductive) $B_L = (1/\omega L)$ is equal to the susceptance (capacitive) $B_C = \omega C$. The phase angle is $\phi = 0°$, and the power factor is unity ($\cos \phi = 1$). The total (supply) current is phase with the input voltage. So, the magnitude of the total current ($\left| (V/R) \right|$) in the circuit is only limited by resistance R. The phasor diagram is shown in Fig. 17.4b.

The magnitude of the current in the inductance, L / capacitance, C (both are equal, as the reactance are equal), is $V(1/\omega_o L) = V \cdot \omega_o C$. This may be termed as the circulating current in the circuit with only inductance and capacitance, the magnitude of which is

$$\left| I_L \right| = \left| I_C \right| = V \sqrt{\frac{C}{L}}$$

substituting the value of $\omega_o = 2\pi f_o$. This circulating current is smaller in magnitude than the input current or the current in the resistance as $\omega_o C = (1/\omega_o L) > R$.

The input current increases as the frequency is changed, i.e. increased or decreased from the resonant frequency ($f > f_o$, or $f < f_o$), i.e. $f \neq f_o$.

In the two cases of series and parallel circuits described earlier, all components, including the inductance, are assumed to be ideal, which means that the inductance is lossless, having no resistance. But, in actual case, specially with an iron-cored choke coil, normally a resistance r is assumed to be in series with the inductance L, to take care of the winding resistance and also the iron loss in the core. In an air-cored coil, the winding resistance may be small and no loss occurs in the air core.

An iron-cored choke coil is connected in parallel to capacitance, and the combination is fed to an ac supply (Fig. 17.5a).
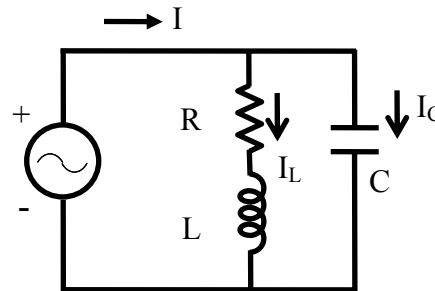

Fig. 17.5 (a) Circuit diagram.

The total admittance of the circuit is

$$Y = Y_1 + Y_2 = \frac{1}{r + j\omega L} + j\omega C = \frac{r - j\omega L}{r^2 + \omega^2 L^2} + j\omega C$$

If the magnitude of the admittance is to be minimum, then

$$\omega C = \frac{\omega L}{r^2 + \omega^2 L^2} \quad \text{or} \quad C = \frac{L}{r^2 + \omega^2 L^2} .$$

The frequency is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{L}{C} - r^2}$$

This is the resonant frequency. The total admittance is $Y\angle 0° = \dfrac{r}{r^2 + \omega^2 L^2}$

The total impedance is $Z\angle 0° = \dfrac{r^2 + \omega^2 L^2}{r}$

The total (input) current is

$$I\angle 0° = \frac{V\angle 0°}{Z\angle 0°} = V\angle 0° \cdot Y\angle 0° = \left(\frac{V}{Z}\right)\angle 0° = (V \cdot Y)\angle 0° = \frac{V \cdot r}{r^2 + \omega^2 L^2}$$

This current is at unity power factor with $\phi = 0°$. The total current can be written as

$$I\angle 0° = I + j0 = I_L\angle -\phi_L + jI_C = I_L\cos\phi_L + j(I_L\sin\phi_L - I_C)$$

So, the condition is $I_C = I_L\sin\phi_l$

where $I_C = \dfrac{V}{X_C} = V \cdot \omega C$; $I_L = \dfrac{V}{\sqrt{r^2 + \omega^2 L^2}}$; $\sin\phi_L = \dfrac{\omega L}{\sqrt{r^2 + \omega^2 L^2}}$

From the above, the condition, as given earlier, can be obtained.

The total current is $I = I_L\cos\phi_L$

The value, as given here, can be easily obtained. The phasor diagram is shown in Fig. 17.5b. It may also be noted that the magnitude of the total current is minimum, while the magnitude of the impedance is maximum.
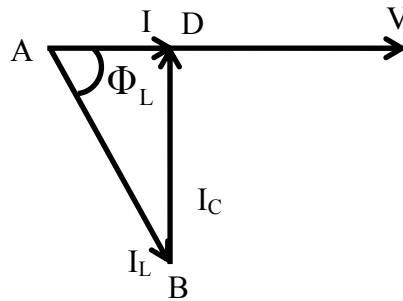
Fig. 17.5 (b) Phasor Diagram

Example 17.2

A coil, having a resistance of 15 Ω and an inductance of 0.75 H, is connected in series with a capacitor (Fig. 17.6a. The circuit draws maximum current, when a voltage of 200 V at 50 Hz is applied. A second capacitor is then connected in parallel to the circuit (Fig. 17.6b). What should be its value, such that the combination acts like a non-inductive resistance, with the same voltage (200 V) at 100 Hz? Calculate also the current drawn by the two circuits.
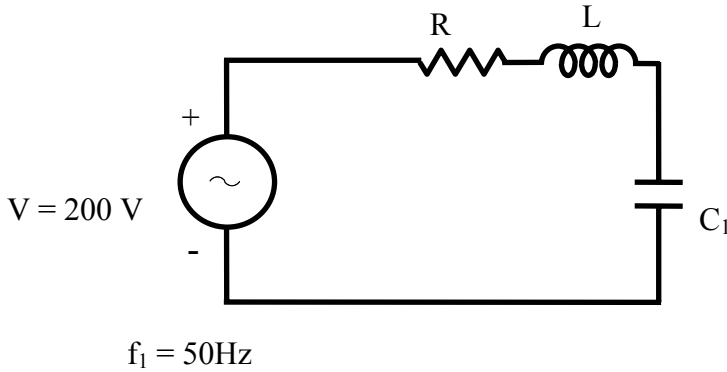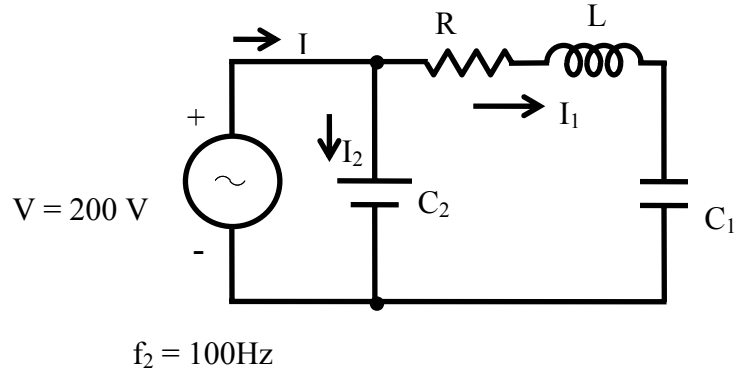
Fig. 17.6 (a) Circuit diagram

f₁ = 50Hz

f₂ = 100Hz

Fig. 17.6 (b) Circuit diagram

Solution

$f_1 = 50$ Hz $\qquad V = 200$ V $\qquad R = 15\ \Omega \qquad L = 0.75$ H

From the condition of resonance at 50 Hz in the series circuit,

$$X_{L1} = \omega_1 L = 2\pi f_1 L = X_{C1} = \frac{1}{\omega_1 C_1} = \frac{1}{2\pi f_1 C_1}$$

So, $C_1 = \dfrac{1}{(2\pi f_1)^2 L} = \dfrac{1}{(2\pi \cdot 50)^2 \times 0.75} = 13.5 \cdot 10^{-6} = 13.5\ \mu F$

The maximum current drawn from the supply is, $I_{max} = V / R = 200/15 = 13.33\ A$

$f_2 = 100$ Hz $\qquad \omega_2 = 2\pi f_2 = 2\pi \cdot 100 = 628.3\ rad / s$

$X_{L2} = 2\pi f_2 L = 2\pi \cdot 100 \cdot 0.75 = 471.24\ \Omega$

$X_{C2} = \dfrac{1}{2\pi f_2 C_1} = \dfrac{1}{2\pi \cdot 100 \cdot 13.5 \cdot 10^{-6}} = 117.8.\ \Omega$

$Z_1 \angle\phi_1 = R + j(X_{L2} - X_{C2}) = 15 + j(471.24 - 117.8) = 15 + j353.44$

$= 353.75\ \angle 87.57°\ \Omega$

$Y_1 \angle -\phi_1 = \dfrac{1}{Z_1 \angle\phi_1} = \dfrac{1}{15 + j353.44} = \dfrac{1}{353.75 \angle 87.57°} = 2.827 \cdot 10^{-3}\ \angle -87.57°$

$= (0.12 - j2.824) \cdot 10^{-3}\ \Omega^{-1}$

$Y_2 = 1/Z_2 = j(\omega_2 C_2)$

As the combination is resistive in nature, the total admittance is

$Y \angle 0° = Y + j0 = Y_1 + Y_2 = (0.12 - j2.824) \cdot 10^{-3} + j\omega_2 C_2$

From the above expression, $\omega_2 C_2 = 628.3 \cdot C_2 = 2.824 \cdot 10^{-3}$

or, $C_2 = \dfrac{2.824 \cdot 10^{-3}}{628.3} = 4.5 \cdot 10^{-6} = 4.5\ \mu F$

The total admittance is $Y = 0.12 \cdot 10^{-3}\ \Omega^{-1}$

The total impedance is $Z = 1/Y = 1/(0.12 \cdot 10^{-3}) = 8.33 \cdot 10^3\ \Omega = 8.33\ k\Omega$

The total current drawn from the supply is

$I = V \cdot Y = V / Z = 200 \times 0.12 \cdot 10^{-3} = 0.024 \; A = 24 \cdot 10^{-3} = 24 \; mA$

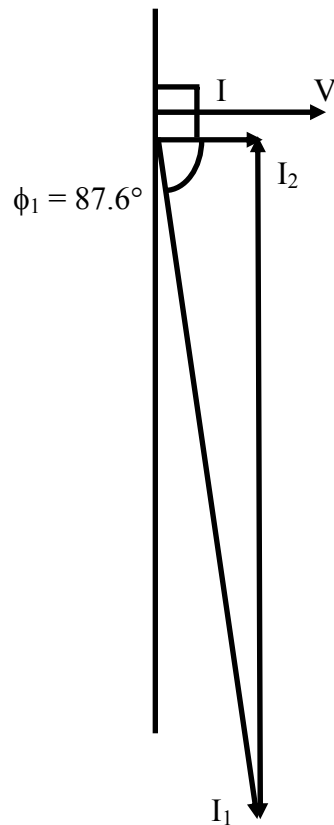The phasor diagram for the circuit (Fig. 17.6b) is shown in Fig. 17.6c.

Fig. 17.6 (c) Phasor diagram

The condition for resonance in both series and parallel circuits fed from single phase ac supply is described. It is shown that the current drawn from the supply is at unity power factor (upf) in both cases. The value of the capacitor needed for resonant condition with a constant frequency supply, and the resonant frequency with constant value of capacitance, have been derived. Also taken up is the case of a lossy inductance coil in parallel with a capacitor under variable frequency supply, where the total current will be at upf. The quality factor of the coil and the bandwidth of the series circuit with known value of capacitance have been determined. This is the final lesson in this module of single phase ac circuits. In the next module, the circuits fed from three phase ac supply will be described.

Problems

17.1 A coil having a resistance of 20 Ω and inductance of 20 mH, in series with a capacitor is fed from a constant voltage variable frequency supply. The maximum current is 10 A at 100 Hz. Find the two cut-off frequencies, when the current is 0.71 A.

17.2 With the ac voltage source in the circuit shown in Fig. 17.7 operating a frequency of f, it was found that I =1.0 ∠0° A. When the source frequency was doubled (2f), the current became I = 0.707 ∠ − 45° A. Find:
   a) The frequency f, and
   b) The inductance L, and also the reactances, X_L and X_C at 2f

17.3 For the circuit shown in Fig. 17.8,
   a) Find the resonant frequency f_0, if R = 250 Ω, and also calculate Q_0 (quality factor), BW (band width) in Hz, and lower and upper cut-off frequencies (f_1 and f_2) of the circuit.
   b) Suppose it was desired to increase the selectivity, so that BW was 65 Hz. What value of R would accomplish this?
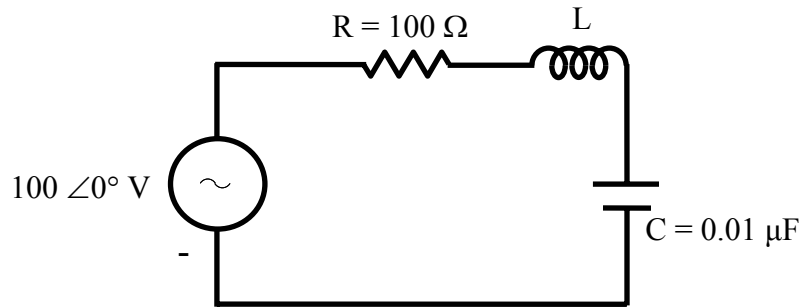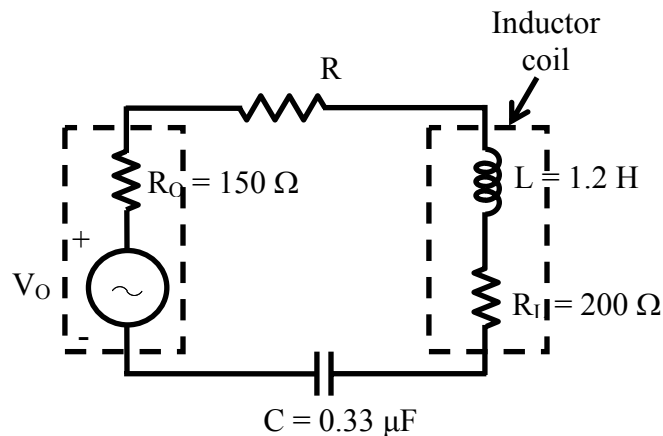

Fig. 17.7

Fig. 17.8

17.4   (a) For the circuit shown in Fig. 17.9, show that the circulating current is given by
$V.\sqrt{C/L}$, if R is small and V is the input voltage.
(b) Find the total current at
(i)   resonant frequency, f_0, and
(ii) at a frequency, $f_1 = 0.9\ f_0$.

17.5   The circuit components of a parallel circuit shown in Fig. 17.10 are R = 60 kΩ, L = 5mH, and C = 50 pF. Find
a)   the resonant frequency, f_0,
b)   the quality factor, Q_0, and
c)   the bandwidth.

Fig. 17.9

Fig. 17.10

List of Figures