Chapter 1
Introduction

The subject of nonlinear control deals with the analysis and the design of nonlinear control systems, *i.e.*, of control systems containing at least one nonlinear component. In the analysis, a nonlinear closed-loop system is assumed to have been designed, and we wish to determine the characteristics of the system's behavior. In the design, we are given a nonlinear plant to be controlled and some specifications of closed-loop system behavior, and our task is to construct a controller so that the closed loop system meets the desired characteristics. In practice, of course, the issues of design and analysis are intertwined, because the design of a nonlinear control system usually involves an iterative process of analysis and design.

This introductory chapter provides the background for the specific analysis and design methods to be discussed in the later chapters. Section 1.1 explains the motivations for embarking on a study of nonlinear control. The unique and rich behaviors exhibited by nonlinear systems are discussed in section 1.2. Finally, section 1.3 gives an overview of the organization of the book.

1.1 Why Nonlinear Control ?

Linear control is a mature subject with a variety of powerful methods and a long history of successful industrial applications. Thus, it is natural for one to wonder why so many researchers and designers, from such broad areas as aircraft and spacecraft control, robotics, process control, and biomedical engineering, have recently showed an active interest in the development and applications of nonlinear control methodologies. Many reasons can be cited for this interest:

• **Improvement of existing control systems:** Linear control methods rely on the key assumption of small range operation for the linear model to be valid. When the required operation range is large, a linear controller is likely to perform very poorly or to be unstable, because the nonlinearities in the system cannot be properly compensated for. Nonlinear controllers, on the other hand, may handle the nonlinearities in large range operation directly. This point is easily demonstrated in robot motion control problems. When a linear controller is used to control robot motion, it neglects the nonlinear forces associated with the motion of the robot links. The controller's accuracy thus quickly degrades as the speed of motion increases, because many of the dynamic forces involved, such as Coriolis and centripetal forces, vary as the square of the speed. Therefore, in order to achieve a pre-specified accuracy in robot tasks such as pick-and-place, arc welding and laser cutting, the speed of robot motion, and thus productivity, has to be kept low. On the other hand, a conceptually simple nonlinear controller, commonly called computed torque controller, can fully compensate the nonlinear forces in the robot motion and lead to high accuracy control for a very large range of robot speeds and a large workspace.

• **Analysis of hard nonlinearities:** Another assumption of linear control is that the system model is indeed linearizable. However, in control systems there are many nonlinearities whose discontinuous nature does not allow linear approximation. These so-called "hard nonlinearities" include Coulomb friction, saturation, dead-zones, backlash, and hysteresis, and are often found in control engineering. Their effects cannot be derived from linear methods, and nonlinear analysis techniques must be developed to predict a system's performance in the presence of these inherent nonlinearities. Because such nonlinearities frequently cause undesirable behavior of the control systems, such as instabilities or spurious limit cycles, their effects must be predicted and properly compensated for.

• **Dealing with model uncertainties:** In designing linear controllers, it is usually necessary to assume that the parameters of the system model are reasonably well known. However, many control problems involve uncertainties in the model parameters. This may be due to a slow time variation of the parameters (*e.g.*, of ambient air pressure during an aircraft flight), or to an abrupt change in parameters (*e.g.*, in the inertial parameters of a robot when a new object is grasped). A linear controller based on inaccurate or obsolete values of the model parameters may exhibit significant performance degradation or even instability. Nonlinearities can be intentionally introduced into the controller part of a control system so that model uncertainties can be tolerated. Two classes of nonlinear controllers for this purpose are robust controllers and adaptive controllers.

• **Design Simplicity:** Good nonlinear control designs may be simpler and more intuitive than their linear counterparts. This *a priori* paradoxical result comes from the fact that nonlinear controller designs are often deeply rooted in the physics of the plants. To take a very simple example, consider a swinging pendulum attached to a hinge, in the vertical plane. Starting from some arbitrary initial angle, the pendulum will oscillate and progressively stop along the vertical. Although the pendulum's behavior could be analyzed close to equilibrium by linearizing the system, physically its stability has very little to do with the eigenvalues of some linearized system matrix: it comes from the fact that the total mechanical energy of the system is progressively dissipated by various friction forces (*e.g.*, at the hinge), so that the pendulum comes to rest at a position of minimal energy.

There may be other related or unrelated reasons to use nonlinear control techniques, such as cost and performance optimality. In industrial settings, ad-hoc extensions of linear techniques to control advanced machines with significant nonlinearities may result in unduly costly and lengthy development periods, where the control code comes with little stability or performance guarantees and is extremely hard to transport to similar but different applications. Linear control may require high quality actuators and sensors to produce linear behavior in the specified operation range, while nonlinear control may permit the use of less expensive components with nonlinear characteristics. As for performance optimality, we can cite bang-bang type controllers, which can produce fast response, but are inherently nonlinear.

Thus, the subject of nonlinear control is an important area of automatic control. Learning basic techniques of nonlinear control analysis and design can significantly enhance the ability of a control engineer to deal with practical control problems effectively. It also provides a sharper understanding of the real world, which is inherently nonlinear. In the past, the application of nonlinear control methods had been limited by the computational difficulty associated with nonlinear control design and analysis. In recent years, however, advances in computer technology have greatly relieved this problem. Therefore, there is currently considerable enthusiasm for the research and application of nonlinear control methods. The topic of nonlinear control design for large range operation has attracted particular attention because, on the one hand, the advent of powerful microprocessors has made the implementation of nonlinear controllers a relatively simple matter, and, on the other hand, modern technology, such as high-speed high-accuracy robots or high-performance aircrafts, is demanding control systems with much more stringent design specifications. Nonlinear control occupies an increasingly conspicuous position in control engineering, as reflected by the ever-increasing number of papers and reports on nonlinear control research and applications.

1.2 Nonlinear System Behavior

Physical systems are inherently nonlinear. Thus, all control systems are nonlinear to a certain extent. Nonlinear control systems can be described by nonlinear differential equations. However, if the operating range of a control system is small, and if the involved nonlinearities are smooth, then the control system may be reasonably approximated by a linearized system, whose dynamics is described by a set of linear differential equations.

NONLINEARITIES

Nonlinearities can be classified as *inherent (natural)* and *intentional (artificial)*. Inherent nonlinearities are those which naturally come with the system's hardware and motion. Examples of inherent nonlinearities include centripetal forces in rotational motion, and Coulomb friction between contacting surfaces. Usually, such nonlinearities have undesirable effects, and control systems have to properly compensate for them. Intentional nonlinearities, on the other hand, are artificially introduced by the designer. Nonlinear control laws, such as adaptive control laws and bang-bang optimal control laws, are typical examples of intentional nonlinearities.

Nonlinearities can also be classified in terms of their mathematical properties, as *continuous* and *discontinuous*. Because discontinuous nonlinearities cannot be locally approximated by linear functions, they are also called "hard" nonlinearities. Hard nonlinearities (such as, *e.g.*, backlash, hysteresis, or stiction) are commonly found in control systems, both in small range operation and large range operation. Whether a system in small range operation should be regarded as nonlinear or linear depends on the magnitude of the hard nonlinearities and on the extent of their effects on the system performance. A detailed discussion of hard nonlinearities is provided in section 5.2.

LINEAR SYSTEMS

Linear control theory has been predominantly concerned with the study of linear time-invariant (LTI) control systems, of the form

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} \tag{1.1}$$

with $\mathbf{x}$ being a vector of states and $\mathbf{A}$ being the system matrix. LTI systems have quite simple properties, such as

- a linear system has a *unique equilibrium point* if **A** is nonsingular;

- the equilibrium point is stable if all eigenvalues of **A** have negative real parts, *regardless of initial conditions*;

- the transient response of a linear system is composed of the natural modes of the system, and the general solution can be solved analytically;

- in the presence of an external input $\mathbf{u}(t)$, *i.e.*, with

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \tag{1.2}$$

the system response has a number of interesting properties. First, it satisfies the *principle of superposition*. Second, the asymptotic stability of the system (1.1) implies bounded-input bounded-output stability in the presence of **u**. Third, a sinusoidal input leads to a sinusoidal output of the same frequency.

AN EXAMPLE OF NONLINEAR SYSTEM BEHAVIOR

The behavior of nonlinear systems, however, is much more complex. Due to the lack of linearity and of the associated superposition property, nonlinear systems respond to external inputs quite differently from linear systems, as the following example illustrates.

Example 1.1: A simplified model of the motion of an underwater vehicle can be written

$$\dot{v} + |v|\,v = u \tag{1.3}$$

where v is the vehicle velocity and u is the control input (the thrust provided by a propeller). The nonlinearity $|v|\,v$ corresponds to a typical "square-law" drag.

Assume that we apply a unit step input in thrust u, followed 5 seconds later by a negative unit step input. The system response is plotted in Figure 1.1. We see that the system settles much faster in response to the positive unit step than it does in response to the subsequent negative unit step. Intuitively, this can be interpreted as reflecting the fact that the "apparent damping" coefficient $|v|$ is larger at high speeds than at low speeds.

Assume now that we repeat the same experiment but with larger steps, of amplitude 10. Predictably, the difference between the settling times in response to the positive and negative steps is even more marked (Figure 1.2). Furthermore, the settling speed v_s in response to the first step is *not* 10 times that obtained in response to the first unit step in the first experiment, as it would be in a linear system. This can again be understood intuitively, by writing that

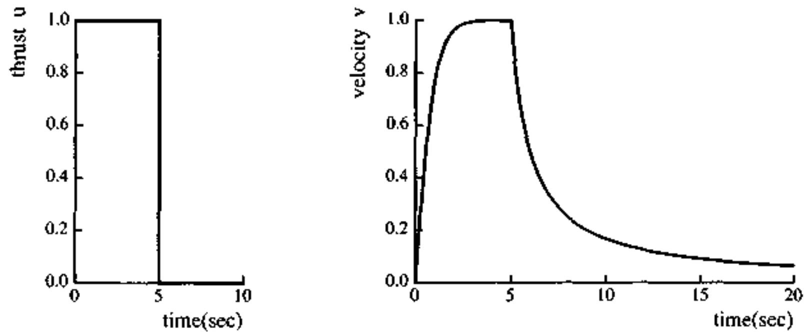

Figure 1.1 : Response of system (1.3) to unit steps

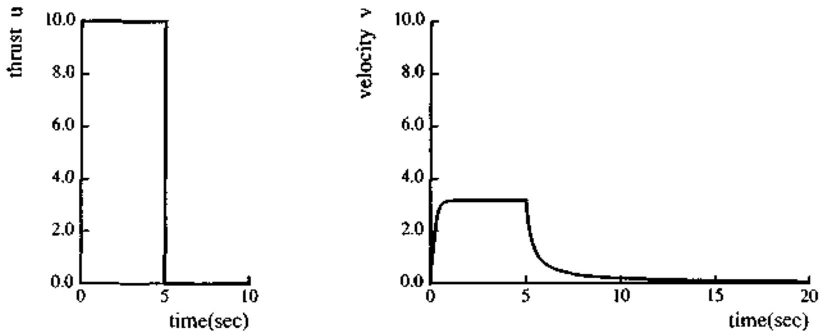

Figure 1.2 : Response of system (1.3) to steps of amplitude 10

$$u = 1 \quad => \quad 0 + |v_s|\, v_s = 1 \quad => \quad v_s = 1$$

$$u = 10 \quad => \quad 0 + |v_s|\, v_s = 10 \quad => \quad v_s = \sqrt{10} \approx 3.2$$

Carefully understanding and effectively controlling this nonlinear behavior is particularly important if the vehicle is to move in a large dynamic range and change speeds continually, as is typical of industrial remotely-operated underwater vehicles (R.O.V.'s). □

SOME COMMON NONLINEAR SYSTEM BEHAVIORS

Let us now discuss some common nonlinear system properties, so as to familiarize ourselves with the complex behavior of nonlinear systems and provide a useful background for our study in the rest of the book.