# *Probability*

# What is Probability?

*Probability is the measure of how likely an event will occur.*

- Probability is the ratio of desired outcomes to total outcomes:

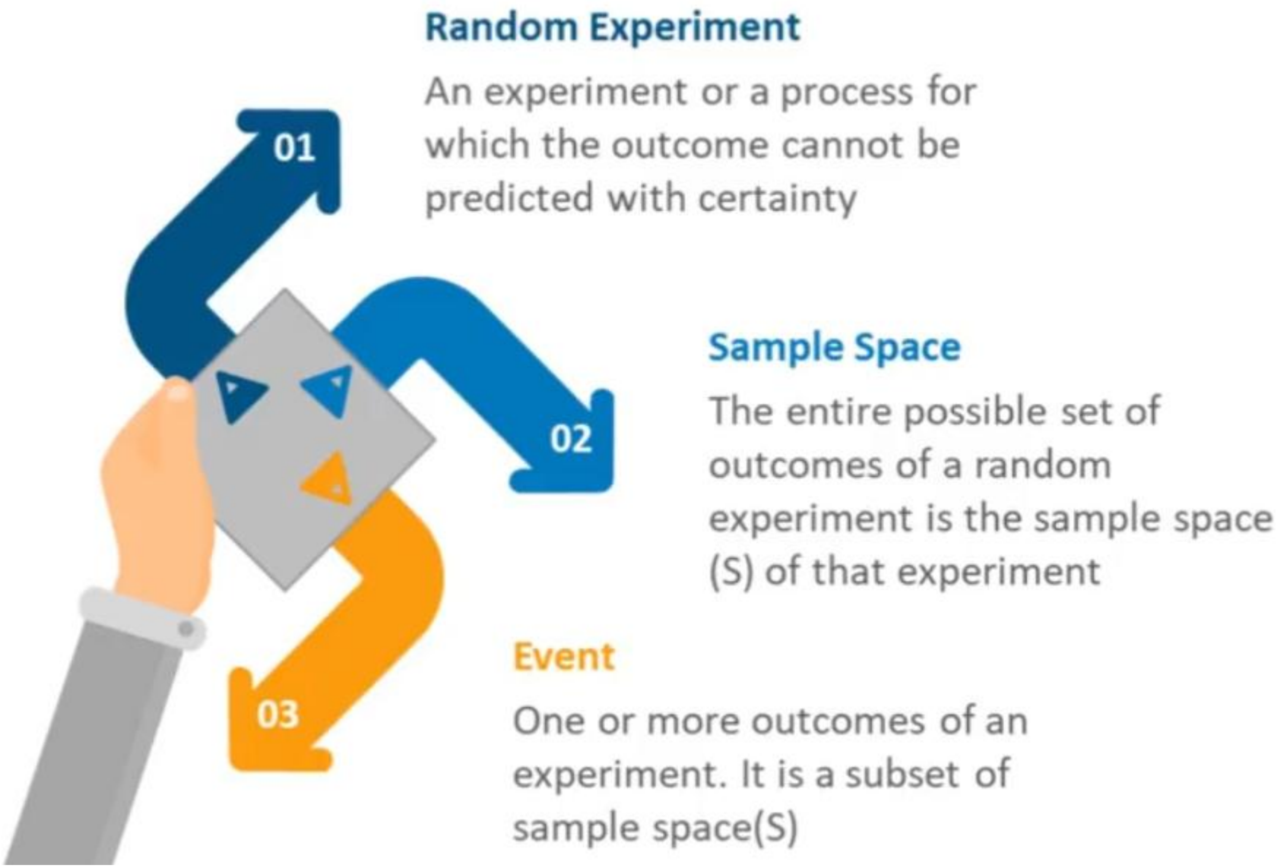
***(desired outcomes) / (total outcomes)***

- Probabilities of all outcomes always sums to 1

Example:

- On rolling a dice, you get 6 possible outcomes
- Each possibility only has one outcome, so each has a probability of 1/6
- For example, the probability of getting a number '2' on the dice is 1/6

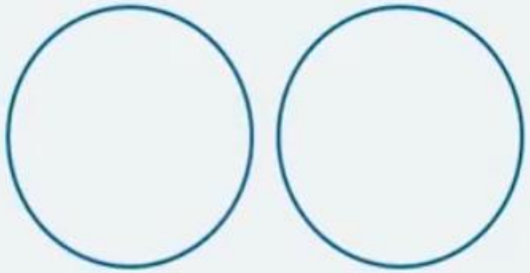
# Terms used in Probability




# Types of events

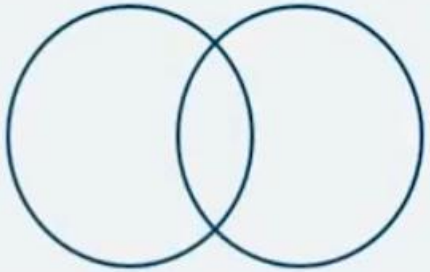
***Disjoint Events*** *do not have any common outcomes.*

- The outcome of a ball delivered cannot be a sixer and a wicket
- A single card drawn from a deck cannot be a king and a queen
- A man cannot be dead and alive



***Non-Disjoint Events*** *can have common outcomes*

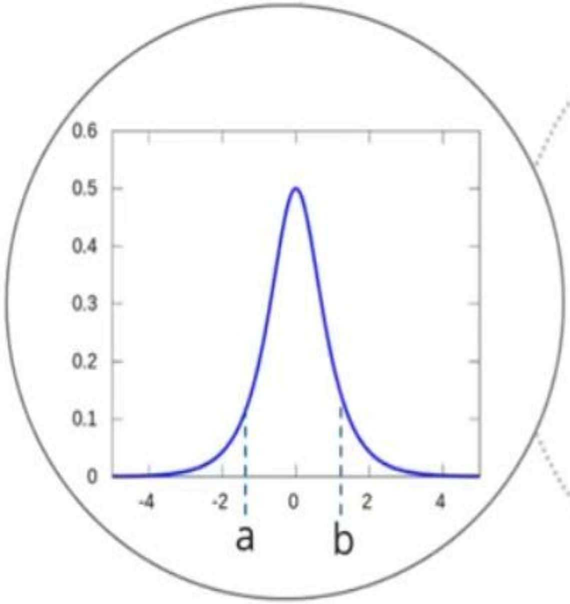
- A student can get 100 marks in statistics and 100 marks in probability
- The outcome of a ball delivered can be a no ball and a six



# Probability Distribution

# Probability Density Function

The equation describing a continuous probability distribution is called a Probability Density Function




**Property 01**

*Graph of a PDF will be continuous over a range*

**Property 02**

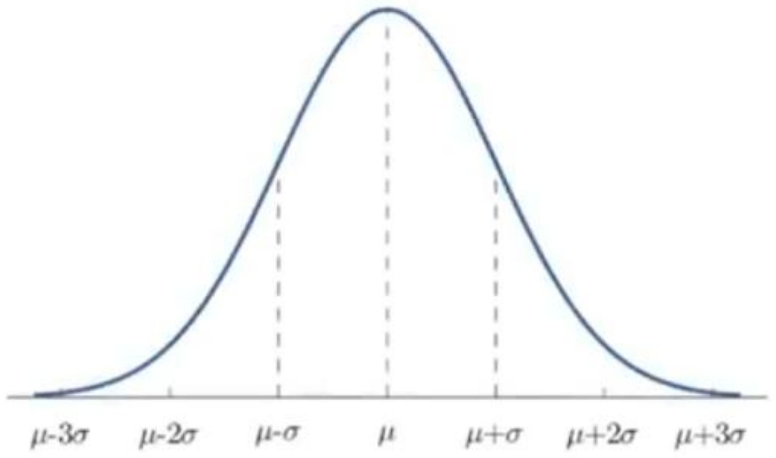
*Area bounded by the curve of density function and the x-axis is equal to 1*

**Property 03**

*Probability that a random variable assumes a value between a & b is equal to the area under the PDF bounded by a & b*

# Normal Distribution

The Normal Distribution is a probability distribution that associates the normal random variable X with a cumulative probability




Y = [ 1/σ * sqrt(2π) ] * e $^{-(x - μ)2/2σ2}$

Where,

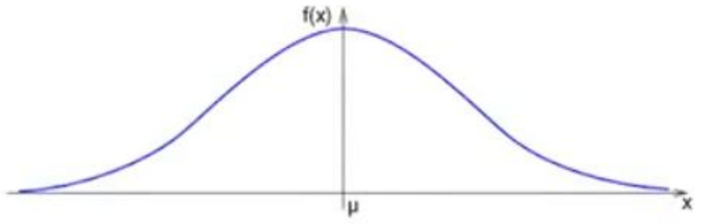
- *X* is a normal random variable
- μ is the mean and
- σ is the standard deviation

**Note:** Normal Random variable is variable with mean at 0 and variance equal to 1

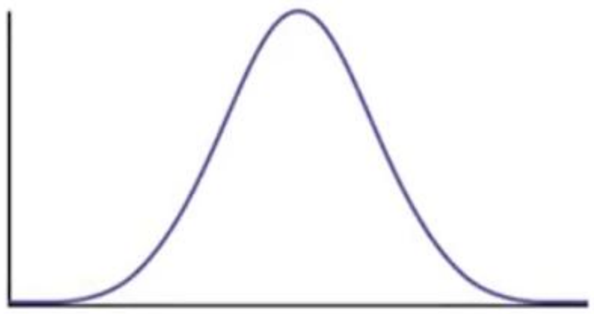
# Standard Deviation and Curve

The graph of the Normal Distribution depends on two factors: the *Mean* and the *Standard Deviation*

- **Mean:** *Determines the location of center of the graph*
- **Standard Deviation:** *Determines the height of the graph*



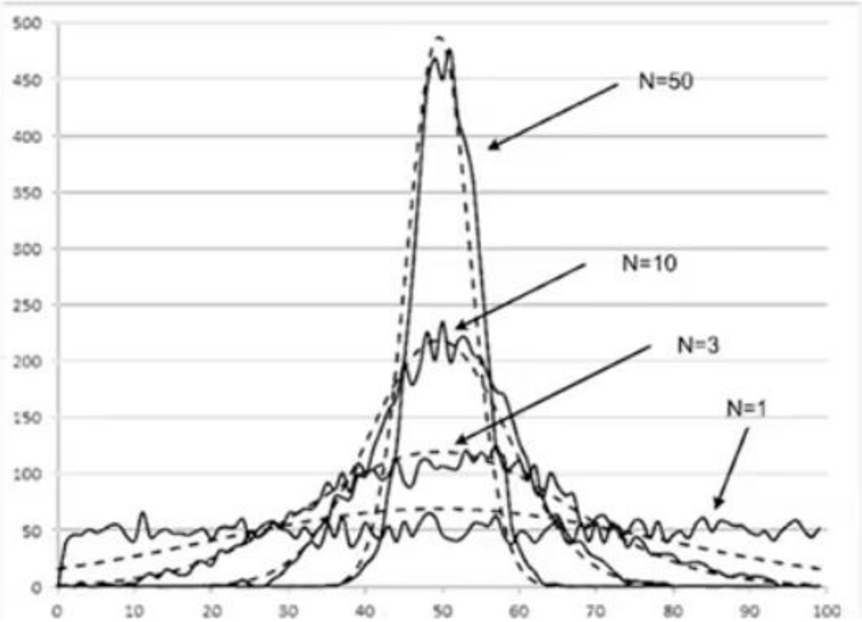

If the standard deviation is large, the curve is short and wide.



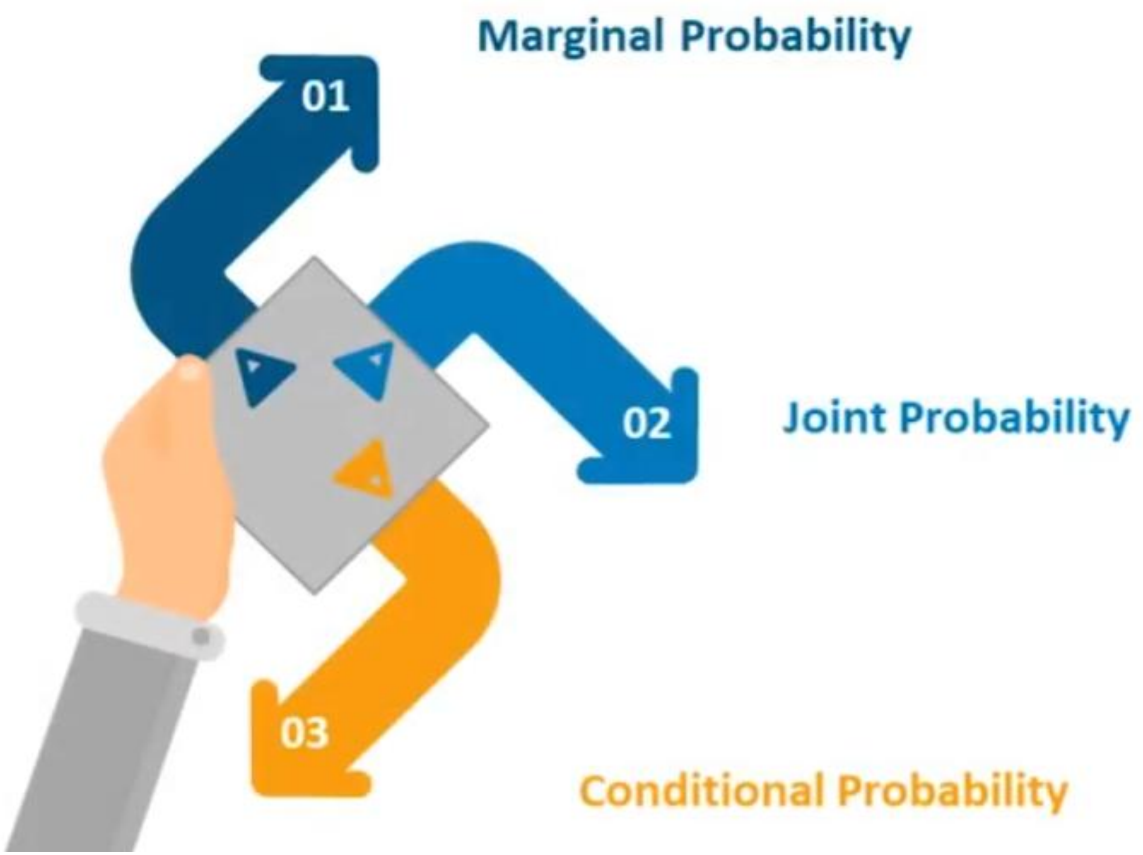
If the standard deviation is small, the curve is tall and narrow.

# Central Limit Theorem

The ***Central Limit Theorem*** states that the sampling distribution of the mean of any independent, random variable will be normal or nearly normal, if the sample size is large enough

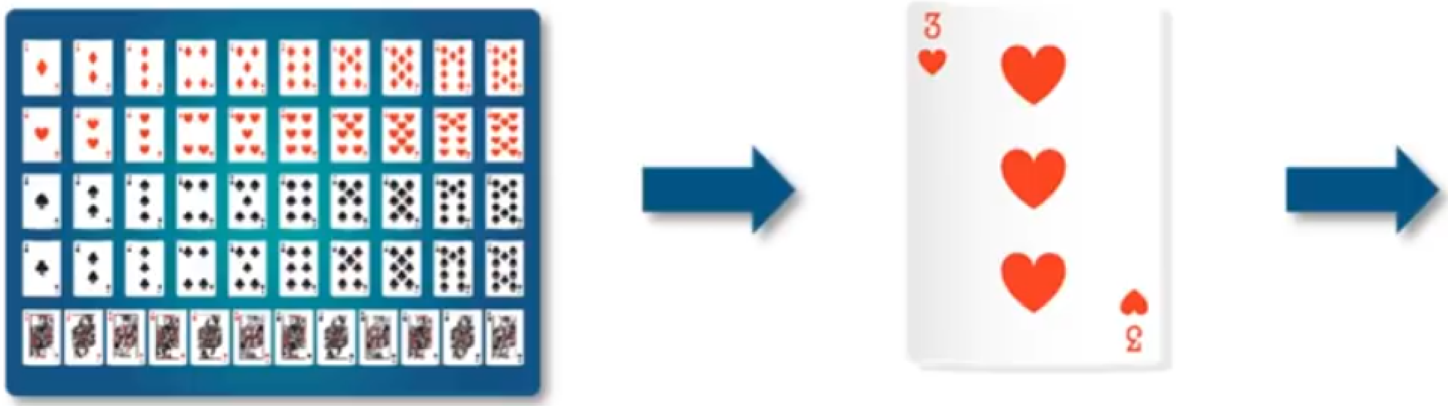


# Types of Probability



# Marginal Probability

*Marginal Probability is the probability of occurrence of a single event.*



Marginal Probability = $\frac{13}{52}$

It can be expressed as: $P(A) = \sum_{i=1}^{k} P(x_i)$

# Joint Probability

*Joint Probability is a measure of two events happening at the same time*

Example: The probability that a card is an Ace of hearts = P (Ace of hearts)
(There are 13 heart cards in a deck of 52 and out of them one in the Ace of hearts)

# Conditional Probability

- *Probability of an event or outcome based on the occurrence of a previous event or outcome*
- *Conditional Probability of an event B is the probability that the event will occur given that an event A has already occurred*

If A and B are dependent events then the expression for conditional probability is given by:
***P (B|A) = P (A and B) / P (A)***

If A and B are independent events then the expression for conditional probability is given by:
***P(B|A) = P (B)***

# Example

| EDUREKA'S TRAINING STUDY |
| --- |
| **TRAINING AND SALARY PACKAGE OF CANDIDATES**<br>Study examines salary package and training undergone by candidates |
| **Sample**: 60 candidates without training and 45 candidates with edureka's training |
| **Task to do**: Assessment of training with salary package |

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
| | Poor Package | 10 | 0 | 10 |
| | Average Package | 40 | 10 | 50 |
| | Good Package | 5 | 30 | 35 |
| | Excellent Package | 0 | 5 | 5 |
| | Total | 60 | 45 | 105 |

# Marginal Probability

*Finding the probability that a candidate has undergone Edureka's training*

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
| | Poor Package | 10 | 0 | 10 |
| | Average Package | 40 | 10 | 50 |
| | Good Package | 5 | 30 | 35 |
| | Excellent Package | 0 | 5 | 5 |
| | Total | 60 | 45 | 105 |

*The probability that a candidate has undergone Edureka's training*

***P(Edu.Training ) = 45 /105 ≈ 0.42***

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
| | Poor Package | 10 | 0 | 10 |
| | Average Package | 40 | 10 | 50 |
| | Good Package | 5 | 30 | 35 |
| | Excellent Package | 0 | 5 | 5 |
| | Total | 60 | 45 | 105 |

# Joint Probability

*Finding the probability that a candidate has attended Edureka's training and also has good package.*

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
| | Poor Package | 10 | 0 | 10 |
| | Average Package | 40 | 10 | 50 |
| | Good Package | 5 | 30 | 35 |
| | Excellent Package | 0 | 5 | 5 |
| | Total | 60 | 45 | 105 |

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
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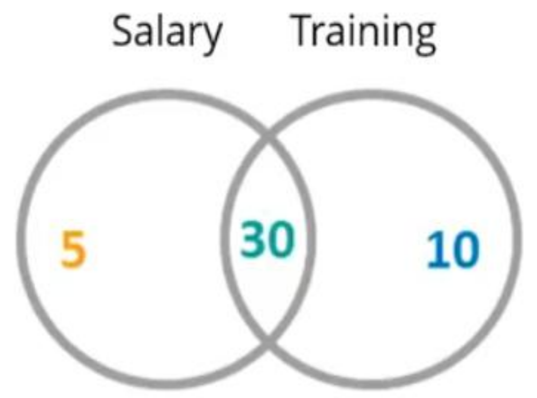


*P (Good Package & Edu.Training ) =*
***30 /105 ≈ 0.28***

# Conditional Probability

*Finding the probability that a candidate has a good package given that he has not undergone training*

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
| | Poor Package | 10 | 0 | 10 |
| | Average Package | 40 | 10 | 50 |
| | Good Package | 5 | 30 | 35 |
| | Excellent Package | 0 | 5 | 5 |
| | Total | 60 | 45 | 105 |

| Results | | Training | | |
|---|---|---|---|---|
| | | Without Edureka Training | With Edureka Training | Total |
| Salary Package obtained by participant | Very Poor Package | 5 | 0 | 5 |
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| | Average Package | 40 | 10 | 50 |
| | Good Package | 5 | 30 | 35 |
| | Excellent Package | 0 | 5 | 5 |
| | Total | 60 | 45 | 105 |

*P (Good Package | Without Edureka) = 5 / 60 ≈ 0.08*

# Bayes Theorem

Shows the relation between one conditional probability and its inverse

P (B|A) is referred to as *likelihood ratio* which measures the probability (given event A) of occurrence of B

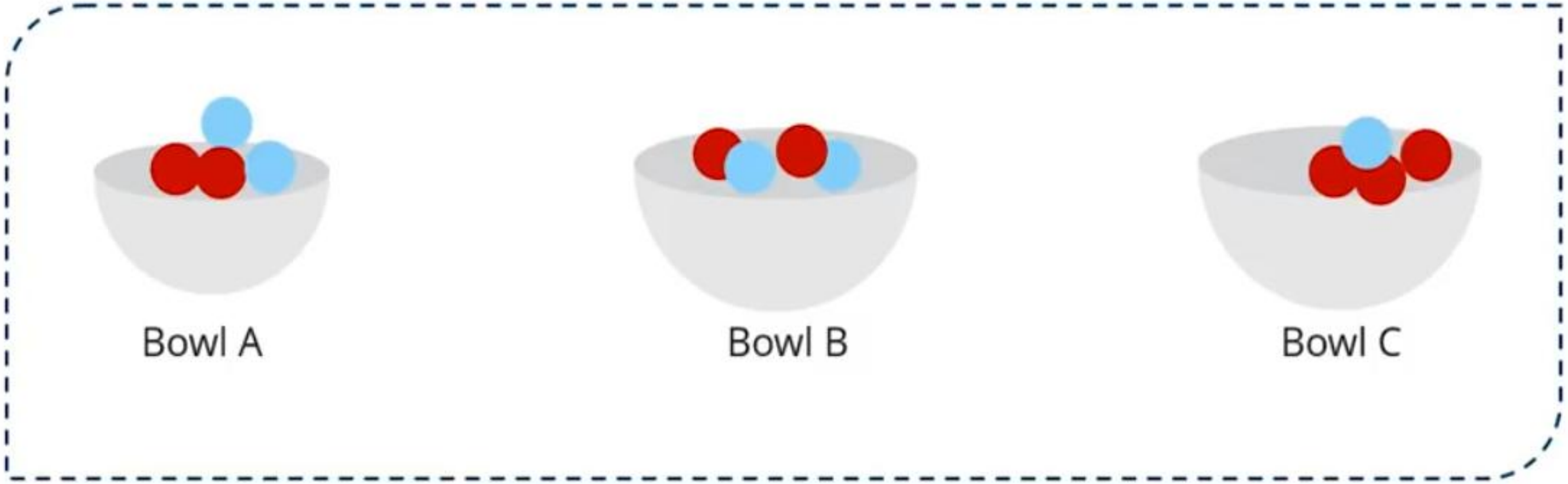
P (A) is referred to as *Prior* which represents the actual probability distribution of A

$$P(A|B) = P(B|A)\,P(A) \,/\, P(B)$$

P (A|B) is referred to as *posterior* which means the probability of occurrence of A given B

# Bayes Theorem Example

*" Consider 3 bowls. Bowl A contains 2 blue balls and 4 red balls; Bowl B contains 8 blue balls and 4 red balls, Bowl C contains 1 blue ball and 3 red balls. We draw 1 ball from each bowl. What is the probability to draw a blue ball from Bowl A if we know that we drew exactly a total of 2 blue balls? "*



- Let *A* be the *event of picking a blue ball* from bag A, and let *X* be the *event of picking exactly two blue balls*
- We want Probability(A|X), i.e. probability of occurrence of event A given X

By the definition of *Conditional Probability*,

$$Pr\ (A|X) = \frac{Pr\ (A \cap X)}{\mathrm{Pr}\ (X)}$$

- We need to find the two probabilities on the right-side of equal to symbolop

Steps to execute the problem

**Step 1:**

First find Pr(X). This can happen in three ways:

(i) white from A, white from B, red from C

(ii) white from A, red from B, white from C

(iii) red from A, white from B, white from C

Next we find $Pr(A \cap X)$.

This is the sum of terms (i) and (ii) above

**Step 2:**