SOLUTION :- RATE OF RETURN METHOD

Discounting Techniques:-

It is a procedure in which the discounting factors are applied systematically to compare various alternatives. The various discounting techniques are

1 **Present worth Method**:- It selects the project with the largest present worth (PW).

The present worth $PW = \sum_{t=1}^{n} (P/F, i\%, n)(B_t - C_t)$

where P = present amount. n = period of analysis
F = future amount. i = discount rate
C_t = cost, B_t = benefit
t = subscripted year.

$(B_t - C_t)$ represent net benefit. If it is constant i.e
$B = (B_t - C_t)$ = constant then

$PW = -k + B(P/A, i\%, n)$ (not derivation but representation)

Certain rules are to be followed while applying the present worth method ~~...~~ for comparing alternatives

Rule a Bring all present worths to the same time base:- i.e. if we have to analyse three projects A, B, C, then their present worth must be brought to the same time base, say 1992.

Rule b Calculate all present worths by using same interest rate.

Rule c Calculate all present worths assuming the same period of analysis for all the projects i.e use one common period of analysis for A, B and C. In this case, we shall come across extended periods for some projects and unused life periods for some projects.

ule d Chose all alternatives having a positive present worth. Neglect the negative values.

ule e ~~[illegible]~~ In a set of mutually exclusive alternatives, chose the alternative with the largest P.W.

ule f If the benefits for two alternatives are almost same, then, chose the alternative having least cost.

Note:- The Above rules are used to sort out the final project out of Many

Example:- Consider the following data and select the project according to PRESENT WORTH METHOD.

	Project A	Project B.
Construction cost	40,000,000	25,000,000 (1st stage) 30,000,000 (2nd stage)
operation and maintenance cost (C_t)	160,000 per year for 40 years.	100,000 per year for 20 yr 220,000 per year for 20 yr
Economic life	40 yrs	40 yrs for each stage
Period of Analysis	40 yrs	40 yrs
Annual Benefits (B_t)	2,500,000	2,500,000
Discount rate	5 percent	5 percent.

Solution:- The first three rules of Present worth Method are satisfied by the data regarding same time base, same discount rate, same period of analysis. We have thus to chose only the positive present worth and finally the Largest present worth.

We shall find the series present worth factor here also.

$i = 5\%$, N = period of analysis = 40 years.

$PW = -k + B(P/A, i\%, N)$ where the symbols are explained earlier. n = period of analysis

-ve means cost

$\boxed{PW \text{ of } A} = -40,000,000 - 160,000 (P/A, 5\%, 40) + 2,500,000 (P/A, 5\%, 40)$

Now $(P/A, 5\%, 40) = \dfrac{(1+i)^n - 1}{(1+i)^n i} = \dfrac{\left(1+\frac{5}{100}\right)^{40} - 1}{\frac{5}{100}\left(1+\frac{5}{100}\right)^{40}} = \dfrac{(1.05)^{40} - 1}{0.05(1.05)^{40}}$

$= \dfrac{6.039}{0.05(7.039)} = 17.159$

$\therefore PW = -40,000,000 - 160,000(17.159) + 2,500,000(17.159)$

$\boxed{PW_A = 153,000}$ $-2745440 + 42897500$

1st stage

$\boxed{PW \text{ of } B} = -25,000,000 - 100,000(P/A, 5\%, 20) - 30,000,000(P/F, 5\%, 20) - 220,000(P/A, 5\%, 20) \times$

next 20 yrs $\rightarrow (P/F, 5\%, 20)$

$+ 25,00000(P/A, 5\%, 40)$

Now, $(P/A, 5\%, 20) = \dfrac{(1.05)^{20} - 1}{0.05(1.05)^{20}} = \dfrac{1.653}{0.05 \times 2.653} = 12.46$

$(P/F, i\%, N) = \frac{1}{(1+i)^N}$ (8)

$(P/F, 5\%, 20) = \frac{1}{\left(1+\frac{5}{100}\right)^{20}} = \frac{1}{2.653} = 0.376$

$\therefore PW_B = -25,000,000 - 100,000(12.462) - 30,000,000(0.376)$
$- 220,000(12.46)(0.376) + 2,500,000(17.159)$

$= 4,308,000$

Thus comparing the present worth of project (A) and (B), we see that Project (B) is having the largest +ive present worth and hence should be chosen.

2. Rate of return method:-

The rate of return method is the second method which helps us to chose from various alternatives i.e. it helps us in comparing alternatives.

The rate of return is the discount rate at which the present worth equals zero. The following rules should be followed.

(Rule a) → Compare all alternatives over same period of analysis

(Rule b) → Calculate the rate of return for each alternative. Chose all alternatives whose rate of return exceeds the minimum acceptable value -Reject rest

(Rule c) → Rank the alternatives in order of increasing cost. Calculate the rate of return of the next alternative above the least costly alternative

LAGRANGE MULTIPLIERS--SOLUTIONS

(Lagrange Multipliers :-)

The basic goal of a project is to Maximise the objective function $u(x, y)$ by chosing the best alternative on the production function $f(x, y) = 0$.
We can take the help of a differential calculus to maximise. It can be done by differentiating the objective function with respect to each $(n+m)$ components.

Lagrange Multiplier is an artificial unknown and is used as a coefficient of the production function $f(x, y) = 0$. This is added to the objective function to form an equation of the type

$$L = u(x, y) + \lambda f(x, y)$$ where λ is the Lagrange Multiplier.

L = variable to be maximised.

example:- In a constrained Maximization problem, the objective function is

$$Y = 10ab \quad \text{----} \; (1)$$

and the constraint is

$$5a + b = 200$$

(Production function = constraint equation)

We have to find a, b, and λ

Solution :- → The constraint equation can be written as

$$5a + b - 200 = 0$$

This eqn is to be multiplied by (λ) and added to the objective function. Thus,

$L = 10ab + \lambda(5a+b-200)$ ---- (A)

we shall use partial differentiation with respect to the unknowns a, b and λ. + setting each =n to zero

$\therefore \frac{\partial L}{\partial a} = 10b + 5\lambda = 0$ --- (1)

$\frac{\partial L}{\partial b} = 10a + \lambda = 0$ --- (2)

$\frac{\partial L}{\partial \lambda} = 5a+b-200 = 0$ --- (3)

By solving the three =ns, we get,

$a = 20, \; b = 100, \; \lambda = -200$.

put these values in =n (A) we get:

$Y = \boxed{20,000}$ This is the maximum value of (Y) subject to the given constraint.

We can introduce as many artifical unknowns as are the number of constraint.

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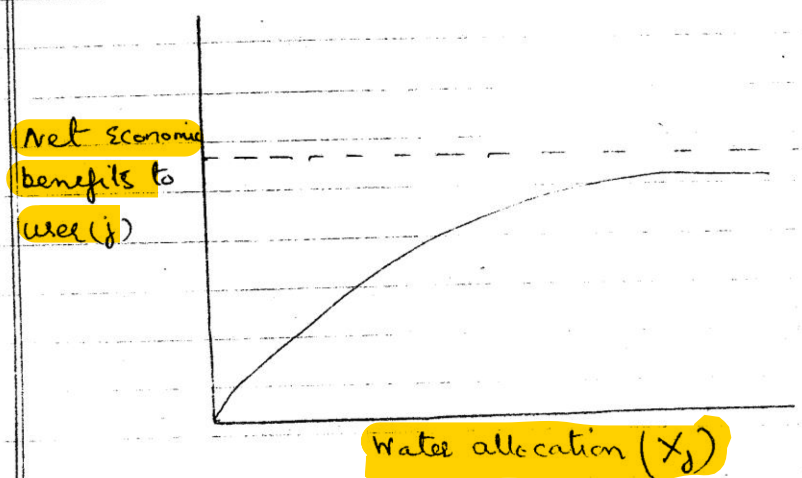
Example 2 :- Consider that a quantity of water = Q has to be alloted to three users denoted by $j = 1,2,3$, where j is the user. The main problem is to give the water quantities x_1, x_2, x_3 to the three users 1,2 and 3 so as to maximise the total net benefit ~~we shall denote x_j in general in place of x_1, x_2, x_3~~ Let us denote the water quantities x_1, x_2, x_3 by a single term x_j where $j = 1,2,3$.

The benefit function is given by $a_j[1 - e^{-b_j x_j}]$ where a_j and b_j are known positive constants.



$x_1 + x_2 + x_3 = Q$. Also the three water allocations x_1, x_2, x_3 are unknown to us. It is also seen that more water supplied will produce more net benefits for the consumer. Thus the constraint equations can be written as

$$\sum_{j=1}^{3} x_j = \theta \quad \text{or}$$

$$\sum_{j=1}^{3} x_j - \theta = 0 \longrightarrow \boxed{\text{constraint equation}} \text{ --- } (1)$$

$$x_j \geq 0, \; j = 1, 2, 3$$

The objective function is given by

$$Y = \sum_{j=1}^{3} \left[a_j (1 - e^{-b_j x_j}) \right] \text{ --- } (2)$$

The procedure is to multiply the production function (constraint = 0) by λ which is the Lagrangian Multiplier and add the product to the objective function to form an equation of the type

$$L = \sum_{j=1}^{3} a_j \left[1 - e^{-b_j x_j}\right] - \lambda\left(\sum_{j=1}^{3} x_j - \theta\right) \quad \text{-----} \; (3)$$

There are four unknowns here. one is λ and rest are the three x_j i.e x_1, x_2, x_3. Differentiating the above eqn with respect to these unknowns, we get

$$\left.\begin{aligned}
\frac{\partial L}{\partial x_1} &= a_1 b_1 e^{-b_1 x_1} - \lambda &= 0 \\
\frac{\partial L}{\partial x_2} &= a_2 b_2 e^{-b_2 x_2} - \lambda &= 0 \\
\frac{\partial L}{\partial x_3} &= a_3 b_3 e^{-b_3 x_3} - \lambda &= 0 \\
\frac{\partial L}{\partial \lambda} &= x_1 + x_2 + x_3 - \theta &= 0
\end{aligned}\right\} \quad (4)$$

The above eqns are the necessary conditions for a local maximum or minimum. The optimal solution to this problem is found by solving for each x_j in terms of λ.

$$\therefore \; x_j = \frac{1}{b_j}\left[-\log\left(\frac{\lambda}{a_j b_j}\right)\right]$$

If we put value of x_j in $= n$ (1) which is the constraint $= n$ we get

$$\theta = \sum_{j=1}^{3} \frac{1}{b_j}\left[-\log\left(\frac{\lambda}{a_j b_j}\right)\right]$$

or $$\theta = \sum_{j=1}^{3} \frac{-\log \frac{\lambda}{a_j b_j}}{b_j} = -\sum_{j=1}^{3} \log\left(\frac{\lambda}{a_j b_j}\right)^{1/b_j} \qquad \left[\because \frac{\log x}{2} = \log x^{1/2}\right.$$

and $$\lambda = \left[e^{-\theta}\left(\sum_{1}^{3}(a_j b_j)^{1/b_j}\right)\right]^{\frac{1}{(1/b_1 + 1/b_2 + 1/b_3)}}$$. knowing a_j, b_j, θ we can find λ.

The value of λ has a very important economic meaning.

PROBLEM ON LAGRANGIAN MULTIPLIERS

Example

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Q. Use a Lagrangian Multiplier to find the Maximum value of $y = 10xw - 4w^2$ subject to the constraint $x + w = 15$. What would be the maximum value if the constraint $x + w = 16$.

Soln:- The objective function is

$$y = 10xw - 4w^2.$$

We have to Maximise the above objective function subject to the given constraint.

Let λ represent the Lagrange multiplier

Now $x + w = 15$ or $x + w - 15 = 0$

We shall multiply the constraint equation (production function) by λ and add the product to the objective function.

i.e. $L = 10xw - 4w^2 + \lambda(x + w - 15)$.

Differentiate the above equation partially w.r.t. the three unknowns x, w and λ, we get

$$\frac{\partial L}{\partial x} = 10w + \lambda = 0 \quad \text{--- (1)}$$

$$\frac{\partial L}{\partial w} = 10x - 8w + \lambda = 0 \quad \text{--- (2)}$$

$$\frac{\partial L}{\partial \lambda} = x + w - 15 = 0 \quad \text{--- (3)}$$

from eqn (3) $x + w = 15$

$\therefore w = 15 - x$.

from eqn (1), $\lambda = -10w$. Put this in (2) we get

$10x - 8w + (-10w) = 0$

or $10x - 18w = 0$

or $10x - (15-x)18 = 0$

or $10x + 18x - 270 = 0$

or $\boxed{x = 9.64}$ ✓

$w = 15 - x = 15 - 9.64$

$\boxed{w = 5.36}$ ✓

also $10 \times 5.36 + \lambda = 0$

$\therefore \boxed{\lambda = -53.60}$ ✓

putting the above values in the $\boxed{\text{objective function}}$ ✓

$y = 10xw - 4w^2$

$= 10 \times 9.64 \times 5.36 - 4(5.36)^2$

$= 516.70 - 115$

Max $\boxed{y = 401.7}$

Case II if the constraint is $x + w = 16$

$L = 10xw - 4w^2 + \lambda(x + w - 16)$

$\frac{\partial L}{\partial x} = 10w + \lambda = 0$ --- ①

$\frac{\partial L}{\partial w} = 10x - 8w + \lambda = 0$ --- ②

$$\frac{\partial y}{\partial \lambda} = x + w - 16 = 0 \quad \text{---} (3)$$

from (3), $w = 16 - x$

from (1), $\lambda = -10w$

$\therefore$ from (2), $\cancel{10x - 8(16-x) + \lambda(-10w) = 0}$

or $\cancel{10x =}$ $10x - 8w + \lambda = 0$

or $10x - 8w + (-10w) = 0$

or $10x - 18w = 0$

or $10x - 18(16 - x) = 0$

or $10x - 288 + 18x = 0$

or $28x = 288$

$x = 10.28$

$w = 16 - 10.28$

$w = 5.72$

$\lambda = -10 \times 5.72 = \boxed{-57.2}$ put the values in the objective function

$\therefore y = 10 \times 10.28 \times 5.72 - 4(5.72)^2$

$= 541.75 - 130.87$

max $\boxed{y = 410.88}$