Course:

× limit state Design philosophy.

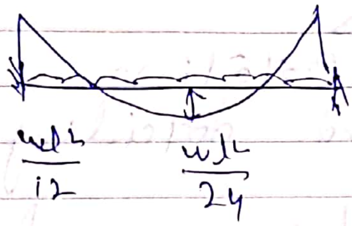
× redistribution of moments in continuous span Beams.

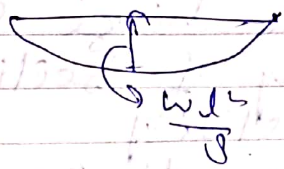
× plastic hinge concept, rotation capacity of sections and detailing for ductility

× yield line theory for slabs - equilibrium and virtual work methods; shrinkage + creep analysis for stresses in compression + flexural members; deflection; shear wall + coupled shear wall; beam column joints; prestressing of continuous beams + portal frames; partial prestressing + circular prestressing.

* WSD & LSD USD

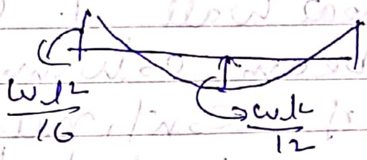
(f.o.s) — collapse, deflection & cracking; partial fact. of s for material & load. — non linear Behavior of str.





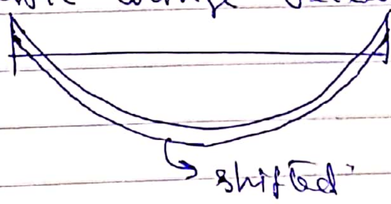
under reinforced → steel will yield & rotation will take place causing redistribution of moment

as



$$\frac{wl^2}{12} - \frac{wl^2}{16} = \frac{wl^2}{48} = \frac{1}{3}\left(\frac{wl^2}{12}\right)$$

initial stresses are less in concrete in underreinforced beams & increase after steel yields to provide rotation. moment diagram shifts. plastic hinge developed.



Depth → governed by deflection criteria

yield line pattern → depends on end conditions.

Reference:

1) A.K. Jain → Limit state Design of Con.
2) Krishna J + Jain O.P., plain + reinforced Con. vol II
3) N. Krishna Raju.

CE 562	Tuesday	wed.	th
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I.S: 456, 13920, 1243 (prestress)

Limit States

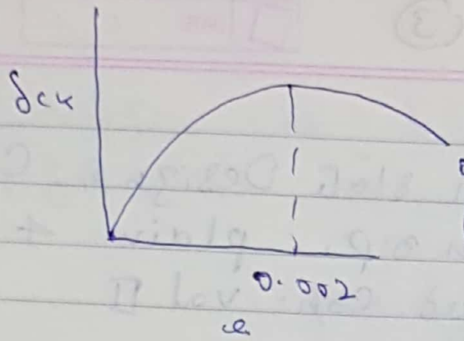
1) deflection
2) cracking
3) Collapse → Bending, shear, axial, Torsion combined, overturning

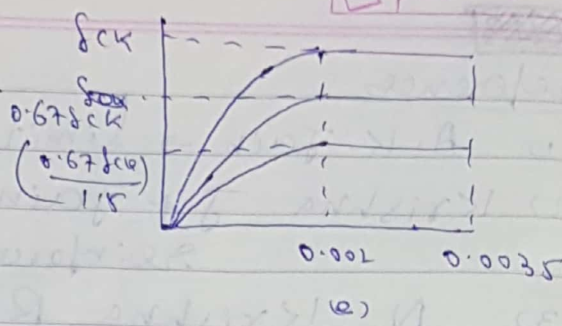
Con. (High st.)	standard	ordinary
M-60		M-10
	M-25	~~M~~ M-15
		M-20
M-80		

Steel:
→ Yield st. (mild steel, well defined yield)
→ 0.2% proof stress → corresponding to strain of 0.002.
→ IS 875 – loads
→ IS 1893 – Seismic loads.

Partial load factors

Combination	D.L	L.L	WL	replace
DL+LL	1.5	1.5	–	WL → E.L
DL+WL	1.5	–	1.5	
Stability DL+WL*	0.9	–	1.5	
DL+LL+WL	1.2	1.2	1.2	

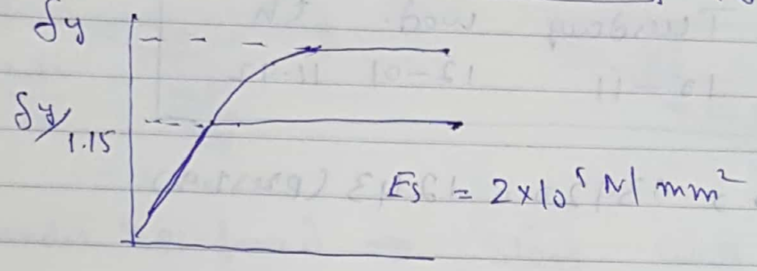




Actual

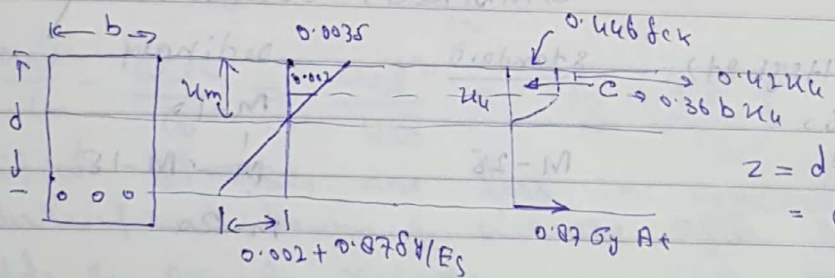
Concrete

Idealized



Steel

02/8/07 :



$z = d - 0.42 x_u$

$= d - \frac{f_y A_t}{f_{ck}\, b}$

Assumptions:

1) plane section normal to the axis remain plane after bending
2) max. strain in concrete at failure is 0.0035
3) max. stress in con. $0.67 f_{ck} / 1.5 = 0.446 f_{ck}$
4) tensile strength of concrete is ignored

i) Balanced Section

Comp. strain in concrete & tension strain in steel reach max. value simultaneously. failure could be sudden & brittle.

→ if steel is less → it will start yielding, con. strain will be less than 0.0035 & steel strain will increase (under-reinforced section) and also con. strain will increase also so that there is some deflection. failure will take place when max. con. strain is reached. & is known as tension failure.

→ over-reinforced sections: con. reaches to max. strain. it is compression failure. The failure is brittle but not as sudden as in case of Balanced sections.

when crack takes place, "I" value decreases & hence deflection increase. So minimum area of steel is provided.

Min. steel = $A_o = 0.87\, bd/f_y$

Max. steel = 4%

max. value of $x_u = x_m$

$$\frac{x_m}{d - x_m} = \frac{0.0035}{0.002 + 0.87 f_y/E_s}$$

from code:

f_y	$x_{u\,max}/d$
250	0.53
415	0.48
500	0.46

$$\frac{x_m}{d} = \frac{0.0035}{0.0055 + 0.87 f_y/E_s}$$

$M_{u\,con} = 0.36 f_{ck}\, b \cdot x_m \cdot z$

$= 0.36 f_{ck}\, b\, x_m (d - 0.42 x_m)$

$M_{u\,steel} = 0.87 f_y A_{st} \cdot (d - 0.42 x_m)$

For Design :

$x_u = 0.87\, f_y A_{st} / 0.36\, f_{ck}\, b$

if $x_u > x_{u\,max}$ (x_m)

section is over-reinforced

∴ failure is governed by con.

$M_{ul} = 0.36\, x_{u\,max} \left(1 - 0.42 \frac{x_{u\,max}}{d}\right) b d^2 f_{ck}$

if $x_u < x_{u\,max}$

Then $M_u = 0.87 f_y \cdot A_{st} \cdot d \left(1 - \frac{f_y A_{st}}{b d f_{ck}}\right)$

Limiting % of Steel

$p_{t\,lim} = \frac{A_{t\,lim}}{bd} \times 100$

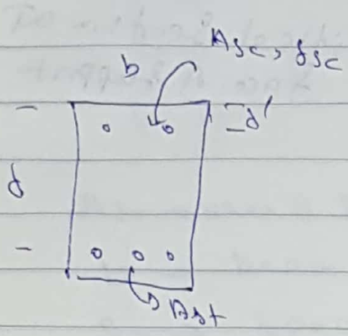
$A_{t\,lim} = \frac{0.36\, \sigma_{ck} \cdot b \cdot x_{u\,max}}{0.87\, \sigma_y}$

$= \frac{0.36\, \sigma_{ck}}{0.87\, \sigma_y} \left(\frac{x_m}{d}\right) \times 100$

f_{ck} \ f_y →	250	415	500
15	1.32		0.76
20	1.76	0.96	0.94
25			
30	2.64		1.13
⋮			

Doubly reinforced section.

$M_F > M_{u\,lim}$
(factored moment)



$M_F - M_{u\,lim} = f_{sc} \cdot A_{sc} (d - d')$

$f_{sc} = 0.0035 (x_{u\,max} - d') / x_{u\,max} \cdot E_s$

Total tension reinforcement $A_{st} = A_{st1} + A_{st2}$

$A_{st1} = A_{st\,lim}$

$A_{st2} = \frac{A_{sc}\, f_{sc}}{0.87 f_y}$

This is for balanced section.

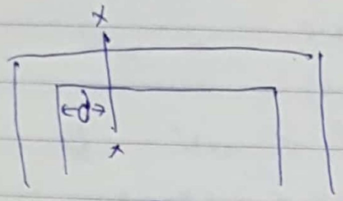
over-reinforced → increase tension steel

under reinforce → increase compression steel

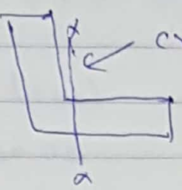
f_{sc} (stress in comp. steel)

f_y	d'/d			
	0.05	0.10	0.15	0.20
250	217	217	217	217
415	355	353	342	329
500	424	412	395	370

Limit state Design for Shear & Torsion:



Beam



critical Section at face of support

Nominal shear stress

$$\tau_v = \frac{V_u}{bd}$$

$$\tau_v \not> \tau_{c\,max}$$

Con. grade	15	20	25	30	35	40 & above
$\tau_{c\,max}$	2.5	2.8	3.1	3.5	3.7	4.0

Slabs → $\tau_v \le \tau_c k$

τ_c → Design shear stress from Table 19.

K for slabs.

over all depth	300	275	250	225	200	175	150 or less
K	1	1.05	1.1	1.15	1.2	1.25	1.3

Usually shear reinforcement is not provided in slabs but if needed

$\tau_v \not> \frac{1}{2}\tau_{c\,max}$ for slabs otherwise revise the Section.

$$\tau_c = \frac{0.85}{6\beta}\sqrt{0.8\,f_{ck}}\left(\sqrt{(1+5\beta)} - 1\right)$$

$$\beta = \frac{0.8\,f_{ck}}{6.89\,p_t} \not< 1$$

$$p_t = \frac{100\,A_t}{b_w\,d}$$

if $\tau_v < \tau_c \rightarrow$ provide nominal reinforcement as given:

$$\frac{A_{sv}}{b \cdot s_v} \geq \frac{0.4}{0.87 f_y}$$

$A_{sv} \rightarrow$ area of shear stirrup in cross section

$b \rightarrow$ beam width

$S_v \rightarrow$ spacing

$f_y \rightarrow$ yield stress of shear reinforcement $\not> 415$

if $\tau_v < \frac{1}{2} \tau_c$ or member of minor ~~flowed~~ structural importance lintel etc.

then we do not provide shear reinforcement.

if $\tau_v > \tau_c \rightarrow$ provide design shear reinf.

$V_u - \tau_c bd = V_{us}$ (shear force to be carried by shear reinforcement)

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v} \quad \text{(for vert. stirrups)}$$

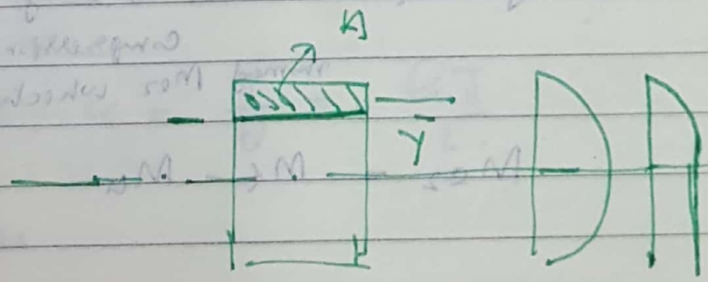
$$= \quad " \quad \times (\sin\alpha + \cos\alpha) \quad \text{for inclined stirrups}$$

$\alpha \not< 45°$

calculate spacing 'S_v' from above expression.

if $\tau_v > \tau_{c\,max} \rightarrow$ increase section size.

$$\frac{VA\bar{y}}{Ib}$$



Torsion → curved beams, balcony, water tanks, curved bridges
(Torsion + shear are considered together)

Equivalent Shear

for Torsion critical section will be 'd' from face of support.

Torsion is max at zero B.M.

T_u = Torsional moment
V_u = Shear

Eqvt. shear:

$$V_e = V_u + \frac{1.6\,T_u}{b}$$

equivalent nominal shear;

$$\tau_{ve} = \frac{V_e}{bd} \not> \tau_{c\,max}.$$

if $\tau_{ve} < \tau_c$ → min shear reinforcement is provided

if $\tau_{ve} > \tau_c$ then long. + Transverse shear reinf. are needed

Eqvt. long. moment (Mel); M_u ; T_u

$$M_{el} = M_u + M_t$$

$$M_t = T_u\left(1 + \frac{D/b}{1.7}\right)$$

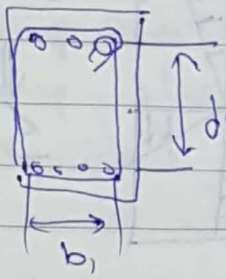
if $M_t > M_u$ → long. reinforcement is needed in compression zone corresponding to moment M_{e2} which is opp. in nature to M_u

$$M_{e2} = M_t - M_u$$

Stirrups → closed hooks

$$A_{sv} = \frac{T_u \cdot S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u \cdot S_v}{2.5\, d_1 (0.87 f_y)}$$

but not less than $\dfrac{(\tau_{ve} - \tau_c)\, b \cdot S_v}{0.87 f_y}$



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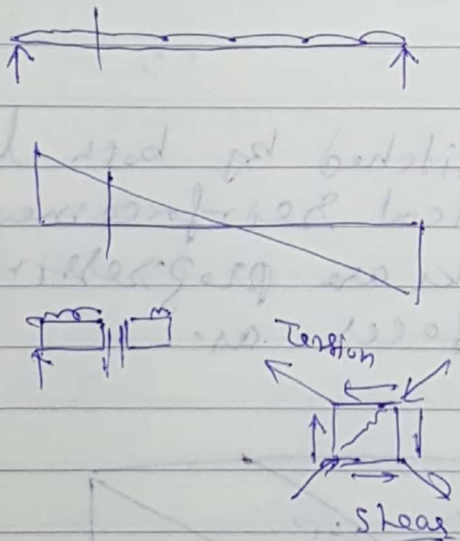
Shear & Torsion

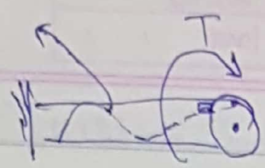


Torsion is not Designed alone it is taken in combination with shear & flexure in concrete.

Torsional stiffness, $K = \frac{T}{\theta} = \frac{GJ}{L}$

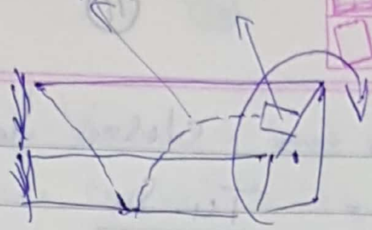
G → shear modulus

Diagonal crack



Diagonal Crack

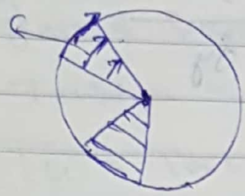
Piece



$$J = \frac{\pi D^2}{32}$$

$$J = \beta x^3 y$$

Shear stress

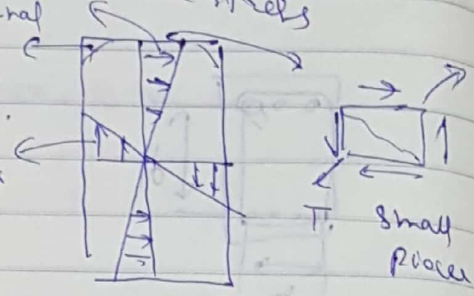


Circular section

Shear stress

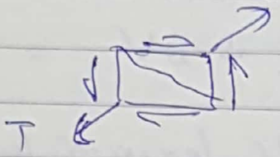
not Torsional stress in extreme corners.

max



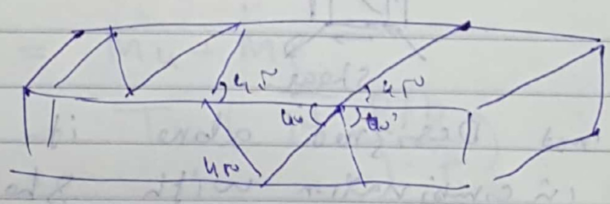
Small pieces

Reactangular section



Small ~~circular~~ element Taken from circular shaft.

Torsion → has to stitched by both longitudinal & ~~st~~ vertical reinforcement. as the ~~cas~~ Cracks are progressing at 45° over all surfaces. as:



eq. shear

$$V_e = V_u + 1.6 \frac{T_u}{b}$$

Shear

Torsion

equivalent shear

Similarly:

$$M_{el} = M_u + M_t$$

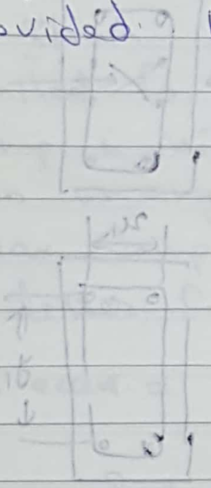
eq. moment.

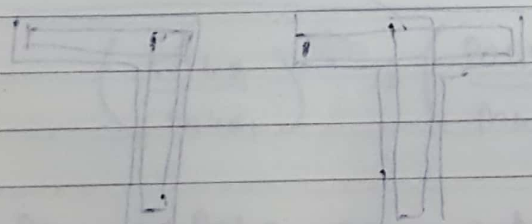
$$M_t = T_u \left(\frac{1 + D/b}{1.7} \right)$$

if $M_t > M_u$ → so we have add moment $M_{e2} = M_t - M_u$.

if then we have to provide longitudinal reinforcement at the top because M_{e2} creates tension.

if $M_t < M_u$; then only comp. in actual comp zone which is taken care by comp. steel provided or min comp steel provided. M_{e2} is also −ve moment.



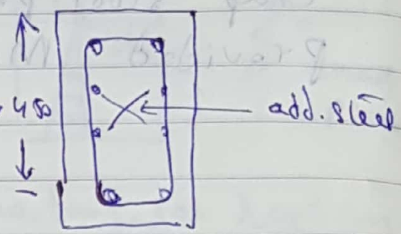


Detailing Provisions

Torsion:

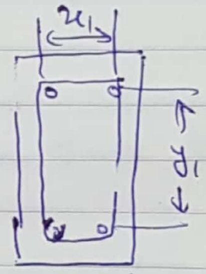
1 long. reinforcement should be placed as near the cor-ners as possible as possible with at-least one long. bar in each corner.

2 if depth exceeds 450 mm ; additional long bars must be provided on the vertical faces equally are of c/s not less than 0.1% of cross section of beam, not greater than width of beam or 300 mm whichever is less.



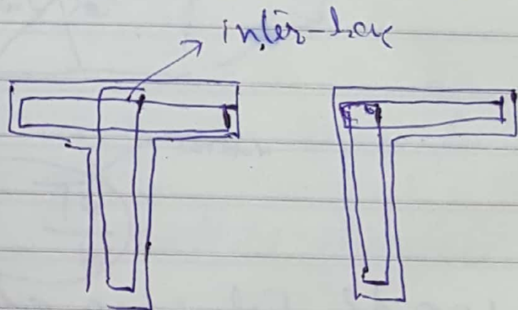
3 Transverse reinforcement consists of closed rect. hoops placed vertically

3 Spacing 'Sv' < x_1
< $(x_1+y_1)/4$
< 300



T & L Beams

if main reinforcement of the slab is parallel to the L/T beam ; transverse reinforcement should be provided in the flange equal to 60% of the area of c/s of main reinforcement
The shear reinforcement cages of the legs should interlock



in T & L Beams if flanges are in tension, then long. Torsional reinf. must be distributed over the flange also. if effective flange width exceeds 1/10 of span then nominal reinforcement must be provided

width = span/10 or eff. flange width whichever is less

Column Design:

1) max. strain due to direct compression is limited to 0.002

if comp. is not uniform but there is no tension on any face then

Max. Comp. ≤ 0.0035 - 0.75 × strain on lesser comp. face.

min eccent. = un-supported length of colum/500 + lateral Dim/30

Sub. to min of 20 mm

Short axially loaded columns

$$P_u = 0.4 f_{ck} \cdot A_c + 0.67 f_y A_{sc}$$

Short Column loaded with axial load and uni-axial moment → Design aids SP-16

Bi-axial Bending + Comp.

$$\left(\frac{M_{ux}}{M_{ux_1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy_1}}\right)^{\alpha_n} \leq 1.0$$

M_{ux} & M_{uy} are actual moments about X & Y axis with the given compressive load.

M_{ux_1} & M_{uy_1} are uniaxial moment capacities with the axial load

α_n depends on P_u / P_{uz}

$P_u = 0.45 f_{ck} + 0.75 f_y A_{sc}$

$\frac{P_u}{P_{uz}} = 0.2$ to $0.8 \rightarrow \alpha_n$ varies linearly from 1 to 2

< 0.2 then $\alpha_n = 1$

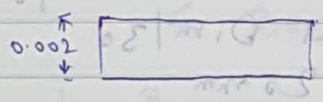
> 0.8 then $\alpha_n = 2$

10/8/05

Short Comp. members:

axially loaded $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$.

for eccentricity up to 0.05 of lateral dimension.

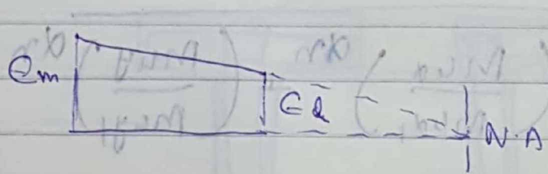


$P = ?$

$M = 0$

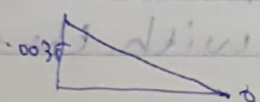
for no eccentricity $e = 0$ N.A → infinity

Comp. strain when axial loaded + flexural $= 0.0035 - 0.75$ of comp. strain on least comp edge.



$e_m = 0.0035 - 0.75\, e_l$

for $e_l = 0 \rightarrow e_m = 0.0035$



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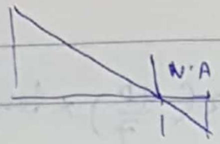
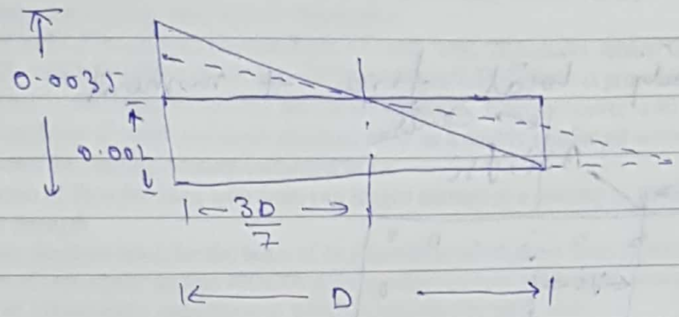
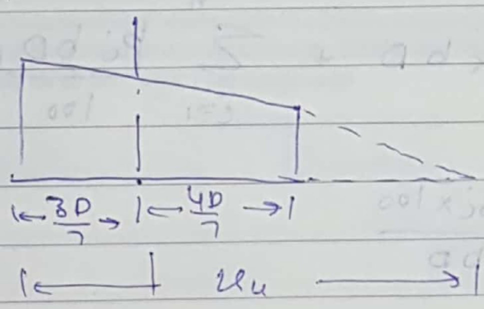


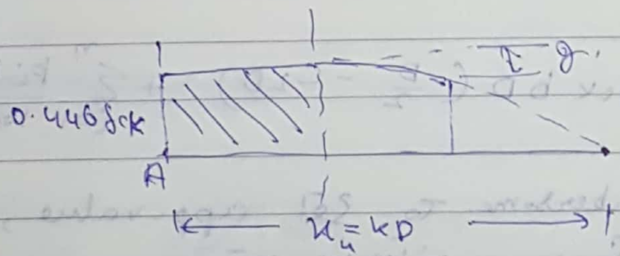
Combine:



General case:



Strain Diagram



Stress diagram

$$g = 0.446 f_{ck} \left[\frac{4/7\,D}{kD - \frac{3D}{7}} \right]^2$$

$$= 0.446 f_{ck} \left(\frac{4}{7k-3} \right)^2$$

area of stress block $= 0.446 f_{ck} D - g/3 \, (4/7\,D)$

$= C_1 f_{ck} D$

when multiplied by 'b' gives total compressive force. (say A)

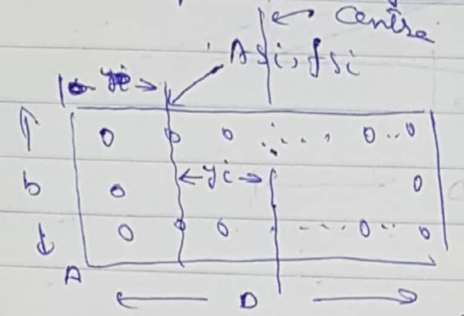
Taking moments about A.

$$= 0.446 f_{ck} D \cdot (D/2) - ?/3 \left(\frac{4D}{7}\right) \times \text{lever arm}$$

$$= M \text{ say.}$$

∴ c.g distance from compressed edge 'A' $= \frac{M}{A}$

For calculating total force + moment.



$$P_u = C_1 f_{ck} b D + \sum_{i=1}^{n} \frac{p_i b D}{100} (f_{si} - f_{ci})$$

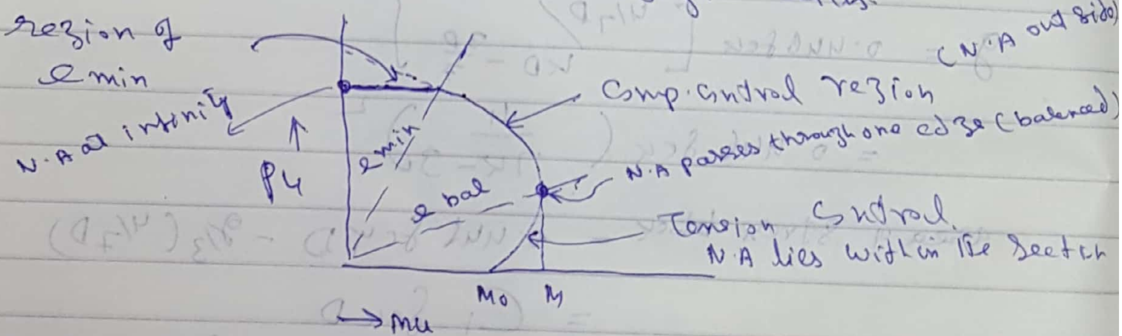
$$p_i = \frac{A_{si} \times 100}{bD}$$

For moment:

$$M_u = C_1 f_{ck} b D \left(\frac{D}{2} - C_2 D\right) + \sum_{i=1}^{n} \frac{p_i b D}{100} (f_{si} - f_{ci}) \times y_i$$

It is cumbersome. To get app. value, nullify f_{ci} + interior steel

But we usually use Design aids.



Interaction Diagram.

Mo

Ginni

(A)

CE-562 Concrete Structures ~~Autumn 2005-6~~

Tut. 1 **Limit State Design**

Use M20 Conc. & Fe415 steel. All given forces and moments are factored values.

1 (a) Design a beam to resist a factored moment of 60 kNm. Assume d/b=2 and p=0.7%. Also calculate the limiting moment and limiting steel area for this section.

(b) A beam of width 200mm and effective depth 350 with 3nos. 20 mm dia. rebars. Calculate Mu.

2 (a) A doubly reinforced beam of width b=260mm and depth D=520mm is provided with 4 nos. 20mm dia. bars in tension and 2 nos. 16 mm dia. bars in compression. Effective cover is 40 mm. Calculate Mu.

(b) Design the beam of width and depth obtained in 1a as a doubly reinforced section for a moment equal to 1.5 times the limiting moment calculated in 1a.

3 (a) The beam of 1b is provided with 8mm two legged stirrups at a spacing of 150mm. Calculate the design shear strength.

(b) Determine the shear reinf. for the beam of 1b required to resist shear force i) 50kN ii) 100) kN.

4 A beam of rectangular section 400x700 deep is subjected to a moment of moment of 100 kNm, a torsion of 15knm and a shear force of 100kN. Calculate the reqd. reinf.

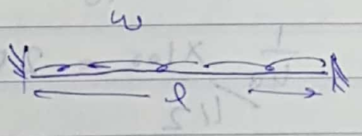
5 (a) Design a short square column to carry a factored axial load of 1500kN assuming 0.8% steel area.

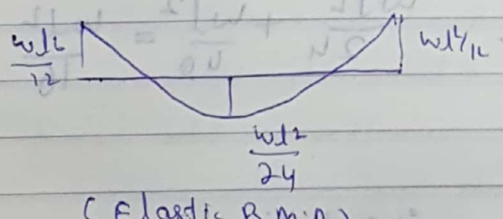
(b) Redesign the above column to carry a moment of 100.KN in addition to the load. Use design aids.

(c) Redesign the above to carry the same axial load along with two moments 100kNm and 75 kNm wrt x and y axes (use design aids)

(d) Design a slender column of cross-section 20x30 effective lengths Lex and Ley = 4m. load =400 kN and moment in longer and shorter directions are 15kNm and 10kNm. Reinf. is to be distributed equally on all four sides. (use design aids)

Moment redistribution in Beams.

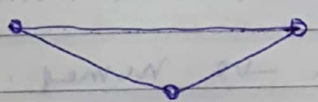




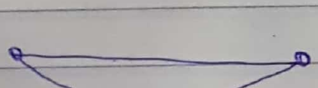
(Elastic B.M.D)

lets assume that supports have lesser moment capacity. i.e $wl^2/16$ not $wl^2/12$

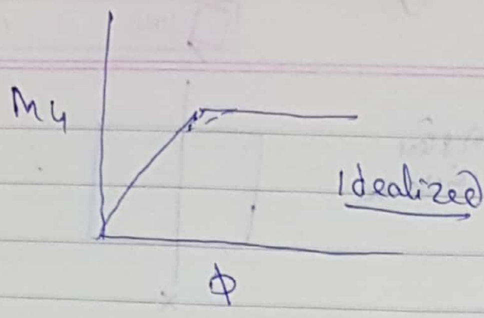
in other word if load increases. Beam will fail if it is balanced; if it is under-reinforced; hinge will form at support, increasing load capacity; then further increase will cause hinge formation at centre, hence failure, in a statically Determinate structure there is no redistribution capacity



Balanced.

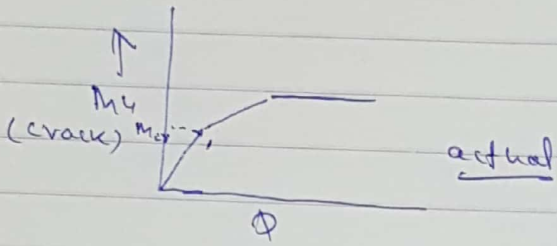


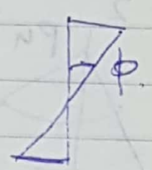
underreinforced



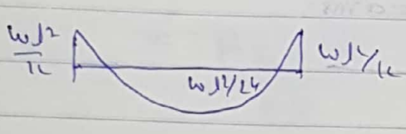
$$\frac{f}{y} = \frac{M}{I} = \frac{E}{R}$$

$$\phi \propto \frac{1}{R}$$



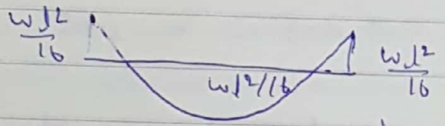


1. Code permits up 30% redistribution of moments in beam
2. In case of frame of over 4 storeys; if the lateral stability of the structure depends on the frame; then only 10% redistribution of moments is permitted.



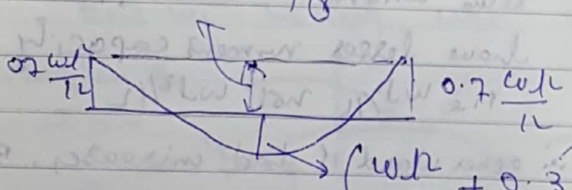
Reduction : $\frac{wl^2}{12} - \frac{wl^2}{16} = \frac{wl^2}{48}$

$\% \quad \frac{1/48}{1/12} \times 100 = 25\%$



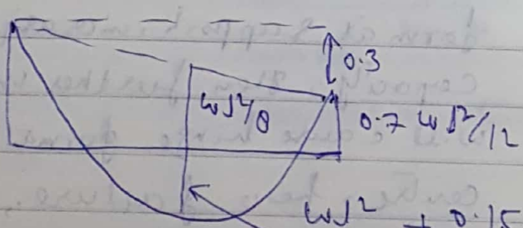
25% reduction

$$\frac{wl^2}{24} + \frac{wl^2}{48} = \frac{wl^2}{16}$$



30% reduction

$\left(\frac{wl^2}{24} + 0.3\frac{wl^2}{12}\right)$, then the moment becomes more than −ve moment.



$$\frac{wl^2}{24} + 0.15\frac{wl^2}{12}$$

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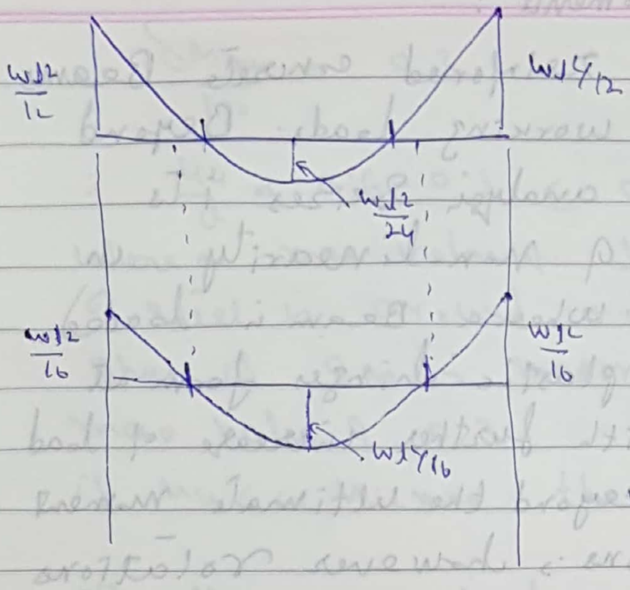

increasing -ve moment capacity reduces the demand of +ve moment requirement. ~~30%~~ not more than 30% is permitted because it will also cause large shifting in point of contraflexure

point of contraflexure changes due to re-distribution