Advanced Structural Analysis

~~Lecture~~: Chapter - I

Types of Structures:

Two types of Structures

1) Skeletal / framed Structures
Beams, plane trusses, plane frames, space frames, space trusses, Grides are the examples.

2) Continuum Structures:
Structures not comprising of individual members but having continuous surface are called as continuum structures.
Flat plate, Shells, folded plate, Dams are the examples.
Shells are curved type of surface structures.

Basic Concepts

All basic structural Analysis methods are founded on the fundamental concepts of

1) Equilibrium
2) Compatibility
3) Force deformation relations.

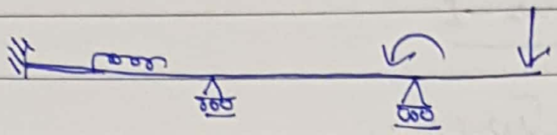
Equilibrium:
The three Governing equations

$$\Sigma F_x = 0$$

$$\Sigma F_Y = 0$$

$$\Sigma M_Z = 0$$

1)



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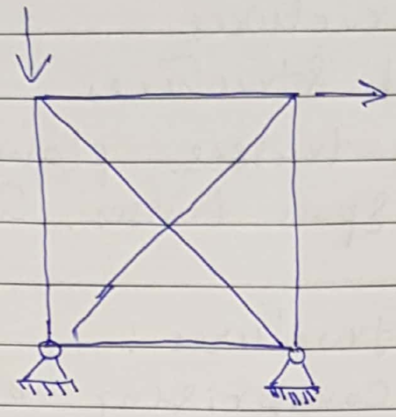

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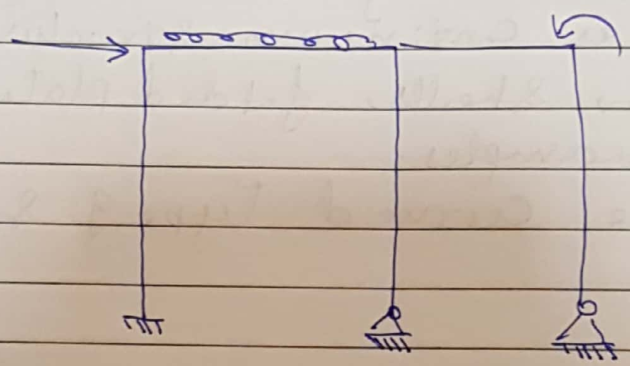

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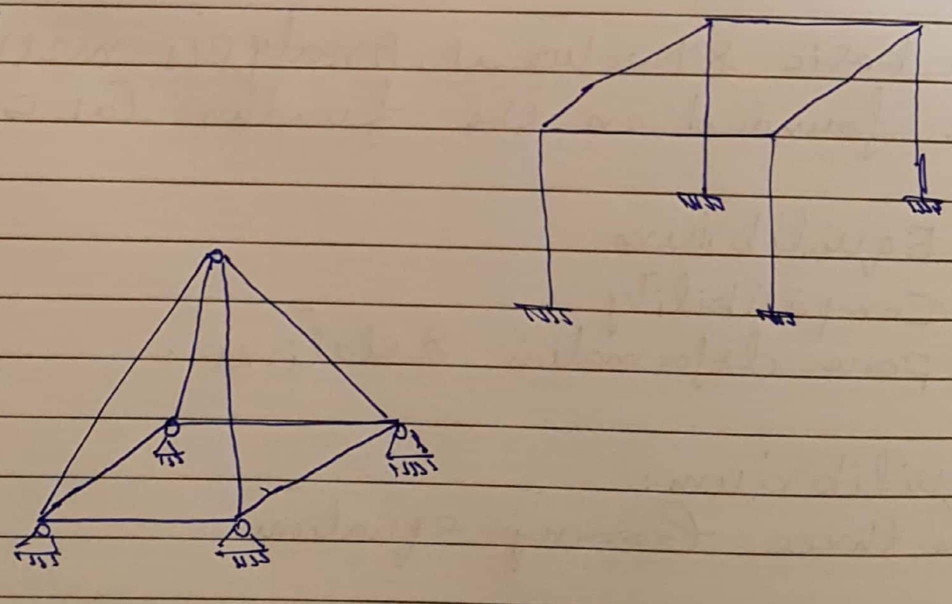


Skeletal / framed structures

Compatibility:

It is essentially a condition that how the structural members must fit together + with their supports.
As we know that equilibrium equations are not enough to determine the un-known forces in a statically indeterminate structure and have to be supplimented by simple Geometrical relations between the Deformations of the structure. Deflection compatibility condition + support compatibility condition are associated with it, e.g. at an intermeadate rigid support of a beam (continuous), there can be no deflection and the rotation on both sides of the support have to be same due to continuity.

Force displacement relations

For deformable bodies, the stress strain relationship of the material of the body is governed by constitutive law.
Equilibrium relate different action i.e forces/stresses while as Compatibility relates deformations i.e displacements / strains.
There are two basic approaches to express these relations
1) Stiffness / displacement approach.
According to this method:

$$F = kD$$

where F + D are the actions and displacements for the member under consideration.

'k' is the stiffness of the member and is defined as the force required to cause unit displacement of the member in the direction of force.

$K = F/D$.

2) Flexibility / force method:

According to this method.

$D = f F$

where 'f' is referred as flexibility of the member and is expressed as the displacement caused by a unit force.

$f = D/P$.

Indeterminacy + Methods of Analysis

The un-knowns of a structural system can be either a set of displacements (Degrees of freedom) or a set of forces (Redundants). When the degrees of freedom are selected as un-knowns. The number of equilibrium equations equal to the kinematic indeterminacy are required to be formed in terms of un-knowns. Displacements are the primary un-knowns. After solving this system for displacements, the displacement of all other points and the internal forces can be determined. The internal forces are axial forces, B.M & S.F. This method is known as displacement method of Analysis.

For example:
1) Unit displacement method.
2) Slope-deflection method.
3) Moment distribution method.
4) Kani's method.
5) Stiffness or matrix displacement method.

When the redundants are un-known, the number of compatibility equations equal to static indeterminacy are required to be formed in terms of un-knowns. After solving for redundants, the internal forces and joint displacements can be determined. This is known as force method of analysis.
For example:
1) Unit load method
2) Energy or Castigliano's theorem
3) Three moment theorem
4) Column analogy
5) Matrix force method / Flexibility method.

Static Indeterminacy
Total static indeterminacy 'd_s' is the sum of external indeterminacy (d_e) plus the internal indeterminacy (d_i)

$$d_s = d_e + d_i$$

$$d_e = NR - NEC$$

NR = Total no of reactions
NEC = No of equilibrium conditions.

$d_i = 3NC$

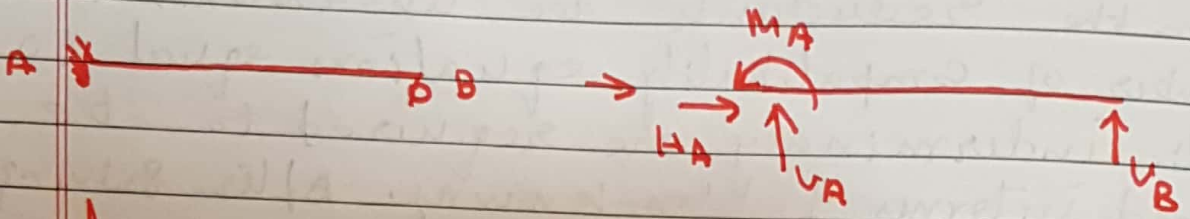
NC = No. of closed spaces.

$\therefore d_s = d_e + d_i$

$= (NR - 3NEC) + 3NC$.

e.g: Consider the beams below:

①

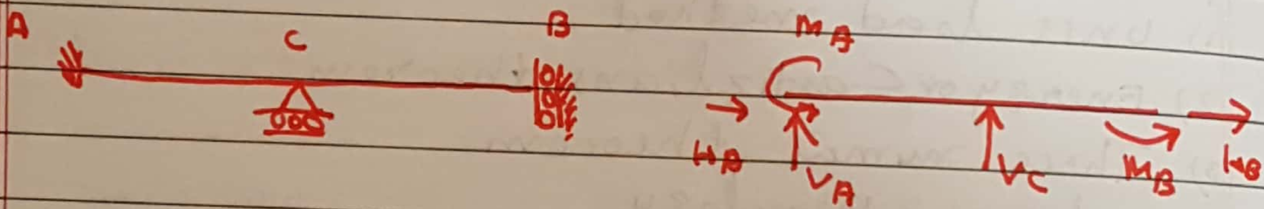


$d_e = 4 - 3 = 1$

$d_i = 0$ (No closed spaces)

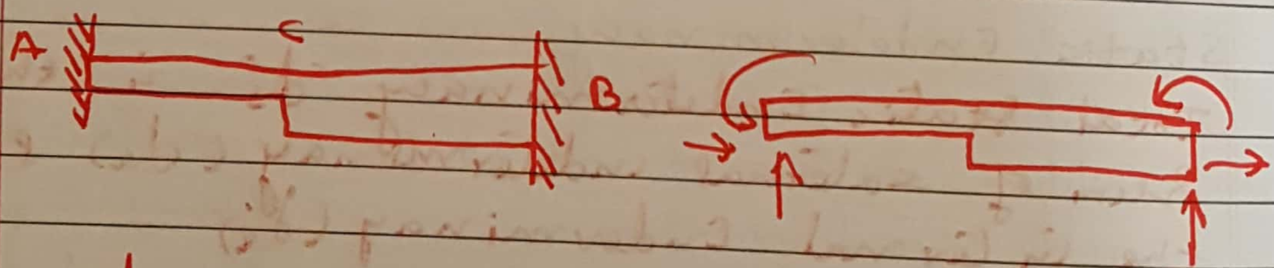
$\therefore d_s = 1 + 0 = 1$

②



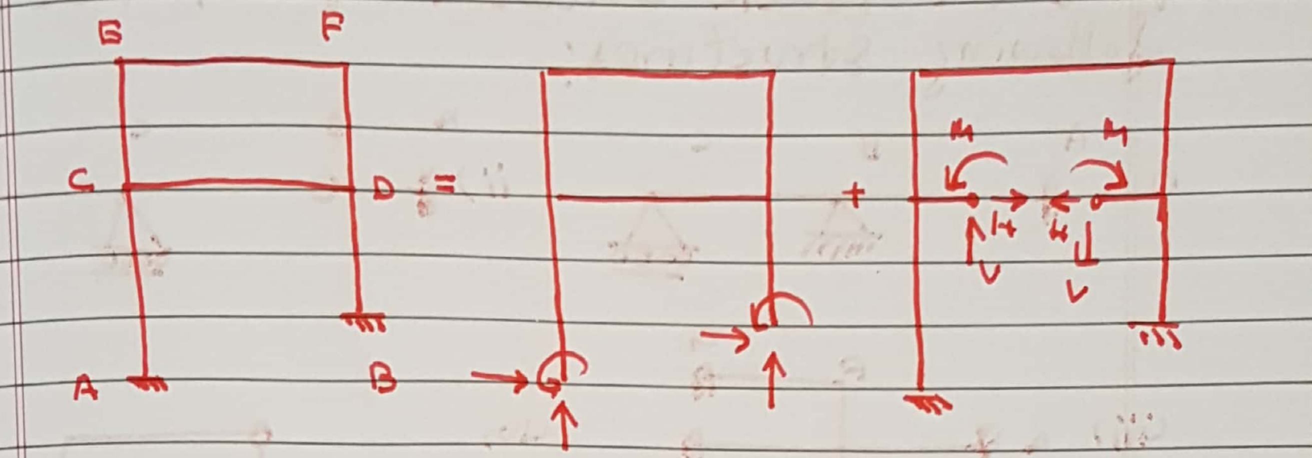
$d_s = 6 - 3 = 3$

③



$d_s = 6 - 3 = 3$

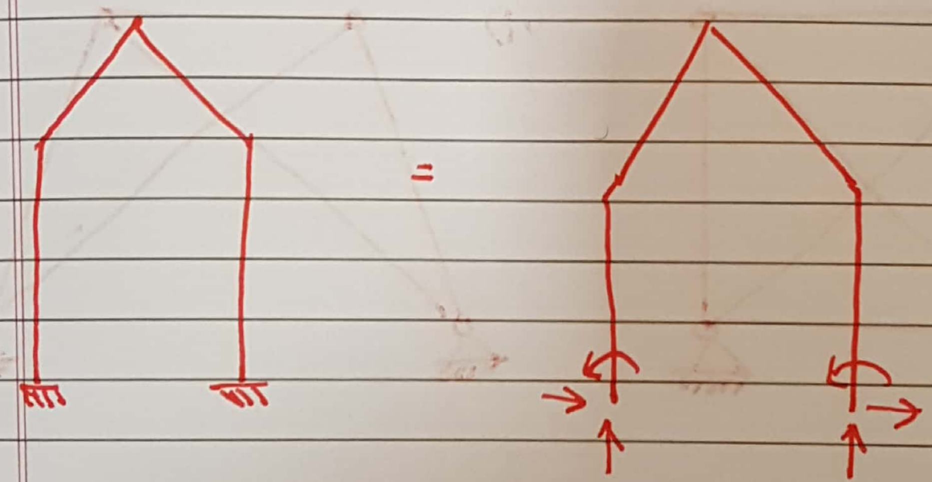
Static Inderminay of Rigid Jointed plan frame



$d_e = 6-3 = 3$

$d_i = 3NC = 3\times1 = 3$ (V, H, M)

$\therefore d_s = 3+3 = 6$

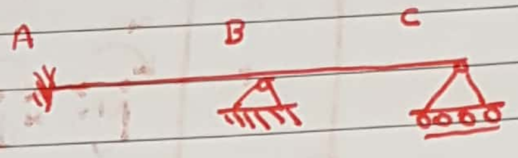


$d_e = 6-3 = 3$

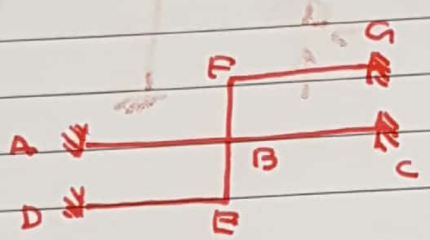
$d_i = 0$

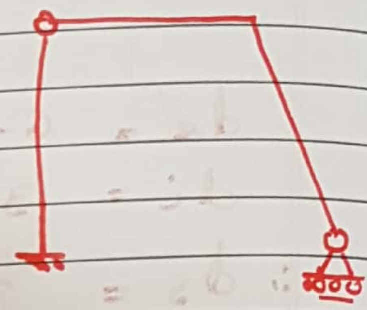
$\therefore d_s = 3+0 = 3$

Exercise-I :-
Find the static Indeterminacy of the following structures:

i) 

ii) 

(iii) 

(iv) 

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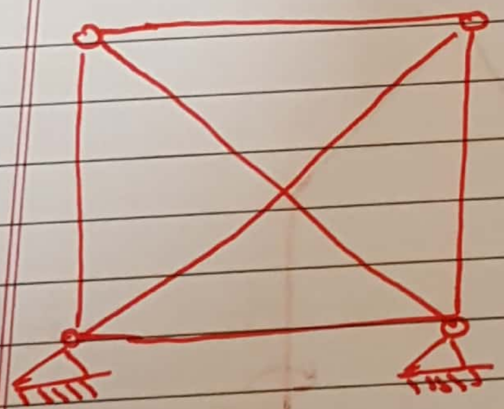
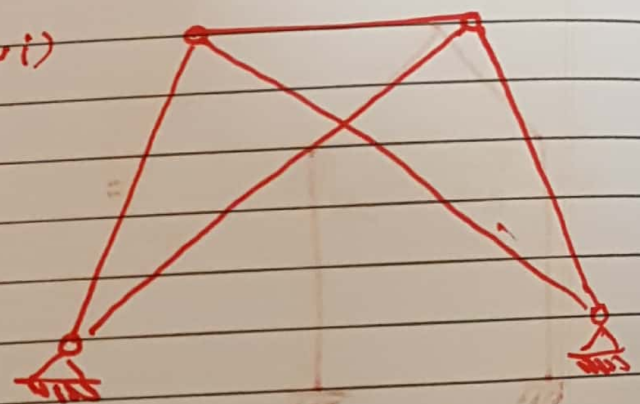


vi) 

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